Free Vibration and Buckling Behaviour of Laminated Composite Panel under Thermal and Mechanical Loading

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Free Vibration and Buckling Behaviour of Laminated Composite Panel under Thermal and Mechanical Loading

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Master of Technology (Research)

by

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Dedicated

To

My Parents

&

Jayesh V. Katariya

Meeta J. Katariya

CERTIFICATE OF APPROVAL

Certified that the thesis entitled "**Free Vibration and Buckling Behaviour of Laminated Composite Panel under Thermal and Mechanical Loading"** submitted by **Katariya Pankajkumar Vaikunthbhai** National Institute of Technology, Rourkela, for award of the degree of Master of Technology (Research) has been accepted by the external examiners and that the student has successfully defended the thesis in the viva-voce examination held today.

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*This is to certify that the thesis entitled, **"Free Vibration and Buckling behaviour of Laminated Composite Panels under Thermal and Mechanical Loading",** being submitted by **Mr. Katariya Pankajkumar Vaikunthbhai** to the Department of Mechanical Engineering, National Institute of Technology, Rourkela, for the partial fulfillment of award of the degree Master of Technology (Research), is an authentic research work carried out by him under my supervision and guidance. This thesis in my opinion, is worthy of consideration for award of the degree of Master of Technology (Research) in accordance with the regulation of the institute. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.*

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Abstract

Laminated composites have been used in various industries such as aerospace, mechanical, chemical, space craft and other high performance engineering applications. This in turn created the requirement of analysis of these structures/structural components through mathematical, experimental and/or simulation based model for accurate design and subsequent manufacturing. These structures are exposed to large acoustic, vibration, inertia excitation as well as unlike environmental condition during their service life. The elevated thermal loading often changes the original geometry of the panel due to excess deformation and the final structural performance affected greatly. The first mode of vibration/fundamental frequency is always associated with high amplitude and it causes large tension and/or compression which leads to fatigue of the structural component. Therefore, the vibration analysis of laminated structures made-up of composite and/or hybrid materials becomes significant. In general, buckling is the state of geometrical instability of the structure induced by the in-plane thermal/mechanical/thermo-mechanical forces. It is important to mention that, the geometric strain associated with buckling is always nonlinear in nature. In this study a general mathematical model is developed for laminated composite single/doubly curved (cylindrical/ spherical/ hyperboloid/ elliptical) panel in the framework of higher order shear deformation theory. The geometrical distortion of the laminated panels due to in-plane (thermal/mechanical/thermo-mechanical) load have been incorporated through Green-Lagrange nonlinearity to count the exact flexure. The developed mathematical model has been discretised using suitable finite element steps to obtain the sets of algebraic equations for the domain. The equations are solved through a computer code developed in MATLAB environment to obtain the desired solutions. In addition to this, a simulation model have been developed in ANSYS for all different cases and the responses are checked to show the generality of the present developed model. The effects of thickness ratio, aspect ratio, curvature ratio, modular ratio, stacking sequence, number of layer and support condition and the material properties on the vibration and the buckling responses are studied in detail.

Keywords: Laminated panel, HSDT, Green-Lagrange nonlinearity, FEM, vibration, thermal/mechanical buckling, ANSYS, APDL code

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Contents

List of Symbols

Most of the symbols are defined as they occur in the thesis. Some of most common symbols, which are used repeatedly, are listed below:

x, y, z	Co-ordinate axis
u, v and w	Displacements corresponding to x, y and z directions, respectively
u_0, v_0 and w_0	In-plane and transverse displacements of a point (x, y) on the mid-plane
θ_x, θ_y and θ_z	Rotations of normal to the mid-plane
ϕ_x, ϕ_y, ψ_x and ψ_y	Higher order terms of Taylor series expansion
R_x, R_y	Principal radii of the curvatures of the shell panel
E_1, E_2 and E_3	Young's modulus
G_{12}, G_{23} and G_{13}	Shear modulus
υ_{12}, υ_{23} and υ_{13}	Poison's ratios
a, b and h	Length, width and thickness of the shell panel
$\{\varepsilon_l\}$ and $\{\varepsilon_{nl}\}$	Linear and nonlinear strain vectors
$\{\sigma\}$	Stress vector at mid-plane
$\{\delta\}$	Displacement vector
$[\bar{Q}]$	Transformed reduced elastic constant
$[B_l]$	Linear strain displacement matrix

$[D]$	Rigidity matrix
$[K_S]$	Linear stiffness matrix
$[K_G]$	Geometric stiffness matrix
$[M]$	Mass matrix
F	Global force vector
$U_{\text{S.E.}}$	Strain energy
$[T]$	Function of thickness co-ordinate
α	Thermal expansion co-efficient
ρ	Density of the material
T	Kinetic energy
W	Work done
a/h	Thickness ratio
R/a	Curvature ratio
E_1/E_2	Modular ratio
ϖ	Non-dimensional fundamental frequency
λ_{cr}	Non-dimensional buckling temperature
N_X	Non-dimensional buckling load

Subscript

l	Linear
nl	Nonlinear
S	Symmetric
i	Node number

Abbreviation

CLPT	Classical laminate plate theory
FSDT	First order shear deformation theory
HSDT	Higher order shear deformation theory
APDL	ANSYS parametric design language
SSSS	All edges simply supported
CCCC	All edges clamped
CFFF	One edge clamped and all other edges free
SCSC	Two parallel edges simply supported and other two edges clamped
CSCS	Two parallel edges clamped and other two edges simply supported
SSCC	Two perpendicular edges simply supported and other two edges clamped
CCSS	Two perpendicular edges clamped and other two edges simply supported
Eq.	Equation
GPa	Giga Pascal
DOFs	Degrees of freedom

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CHAPTER 1

INTRODUCTION

1.1 Overview

Nowadays laminated composite shells are used in many structural parts of various modern vehicles, buildings, historical and engineering structures. A shell panel can be defined as curved thin/thick surface. It may be made from a single layer or multilayer of isotropic or anisotropic materials. The shell panel can be classified according to the curvatures such as doubly curved (both principal curvatures are unequal), cylindrical (one of the principal curvature is zero), spherical (both principal curvatures are equal), conical (where one of the curvatures is zero and the other changes linearly with the axial length), and flat panel (both curvatures are zero). Laminated composites are incredibly lightweight, especially in comparison to traditional materials like concrete, metal, and wood. Composite materials are extremely strong especially per unit of volume/weight, low co-efficient of thermal expansion, excellent elastic properties and good corrosion resistant and highly resistant to chemicals. The composites have ability to allow the structural properties to be tailored according to requirements which add to their versatility for high performance engineering applications. A sufficient amount of weight can be dropped by using composites as compared to conventional materials, viz. the new Boeing 787 (Dreamliner), has used 50 percent composite materials dropping its overall weight by 12% (approx.) and added strength and lower weight allow the plane to use less fuel [76]. In order to meet the economic challenges present days it is necessary to manufacture the composite structures on the large scale as it effect on the cost and availability of the composite structure. It is necessary to analyse these components through mathematical and/or simulation based model beforehand for design and manufacturing. The external skins of aircraft/spacecraft/automobile are having panel type of geometry and made of the thin laminated composites. As discussed the structural components of high speed aircrafts, rockets and launch vehicles are subjected to intense loading due to the aerodynamic heating during their service as a result the structural responses such as deformations, buckling and natural frequencies are affected considerably.

The shell panel has significantly higher membrane stiffness than that of the bending stiffness due to which it is capable to absorb a large value of membrane strain energy without excessive deformations. If the shell is loaded in such a way that most of its strain energy is in the form of membrane compression, and if there is a way that this stored-up membrane energy can be converted into bending energy, the shell may fail dramatically in a process called "buckling". Hence, the buckling plays an important role in the design and analysis of the structures. Basically two types of buckling occur in structural member namely, eigenvalue/ bifurcation buckling and non-linear/ limit point buckling [74]. The bifurcation buckling is a form instability in which there is a sudden change of shape of the structure due to the axial compressive/tensile load. However, in the limit point buckling there is no sudden change of shape but it deviates from the primary equilibrium path after reaching the critical load i.e., known as "snap through". It is well known that buckling doesn't mean the failure of the structural components and they are still capable to carry extra amount of load beyond the buckling point without failure.

Today most of the engineering structural components firstly replaced with laminated composite curved/flat panels due to their tailor made properties. The panel structures are known for their high energy absorbing capability. It is also, true that the original geometry of the panel is distorted because of the excess deformation due to induced in-plane thermal/mechanical stress. This in turn affects the stiffness properties of the panel structure. The vibrations with high amplitude cause great tensions and the reduction of life due to fatigue. It is well known that, thin laminated structures are prone to buckle as well as the structural geometry affects the final performances under combined loading.

It is important to mention that, the geometric strain associated with the buckling phenomena is nonlinear in nature. Many researchers have already reported the buckling responses of the laminated structures by taking the geometric nonlinearity in von-Karman sense based on the various classical and shear deformation theories. Hence, to predict the buckling strength of laminated shell panel structures, the mathematical model has to be sufficient enough to incorporate the geometric alteration due to the excess deformation under in-plane loading. The Green-Lagrange type nonlinear kinematics is taken instead of von-Karman type for the actual prediction for a realistic model development to analyse the buckling of laminated structures. However, the studies related to buckling by taking the geometric nonlinearity in Green-Lagrange sense in conjunction with higher order mid-plane kinematics are very few. Hence there is a need to investigate the buckling of the shell panels taking the nonlinearity in Green-Lagrange sense based on the higher order shear deformation theory for more accurate prediction of the responses of the structure. Therefore the stability and vibration analysis of laminated structures made-up of composite materials becomes significant.

To understand the realistic nature of the structural responses of vibration and buckling (thermal/mechanical/thermo-mechanical) of laminated composite curved/flat panel, the mathematical model is proposed to be developed based on a higher order shear deformation theory (HSDT). It is important to mention that the strain displacement relation is considered in Green-Lagrange type to take into account the geometrical nonlinearity arising in the curved panel due to excess thermal and/or mechanical deformation for evaluation of geometric stiffness matrix in buckling case. It is worthy to mention that the structural components made up of laminated composite has created the necessity to model and analyse, it is because of the fact that these problems are not only interesting but also challenging in many front. Hence, the actual predictions of the fundamental frequency and critical buckling load/temperature parameters are necessary to examined and to achieve the same, modelling of these structures is one of the major aspects from a designer's point of view. Parametric study gives the idea regarding effect of geometries, material properties, types of loading of laminated structure and their limiting conditions. In order to introduce the present problem some of the earlier work completed by other researcher have been discussed in the forthcoming section.

1.2 Literature Review

The vibration and buckling (thermal/mechanical/thermo-mechanical) behaviour of laminated composite panels are studied by many researchers to fill the gap by using numerical and analytical methods. From last few decades, laminated composite panels are modelled based on various theories to capture the true flexure of the structures under the influence various types of combined loading. The analysis of plate and shell structures are mainly based on the three theories:

- The classical laminate plate theory (CLPT)
- The first-order shear deformation theory (FSDT)
- The higher order shear deformation theory (HSDT)

The classical laminate plate theory and the first-order shear deformation theories have been used by many researcher in past due to their many advantages and these theories are also encounter with one major lacuna like shear correction factor when analysing for thin laminated structure. Hence, to overcome from the lacuna of above discussed theories higher order theories have been developed to model the mid-plane kinematic deformation correctly.

However, the stress resultant involved in that are difficult to interpret physically and need much more computational effort. In first order shear deformation theory and higher order shear deformation theory the effect of transverse shear deformation may be essential in some cases, whereas it is neglected in classical laminate plate theory due to the Kirchhoff hypothesis. The CLPT is based on the Kirchhoff hypothesis that straight lines normal to the undeformed mid plane remain straight and normal to the deformed mid plane and do not undergo stretching in the thickness direction which is presented in Figure 1.1.

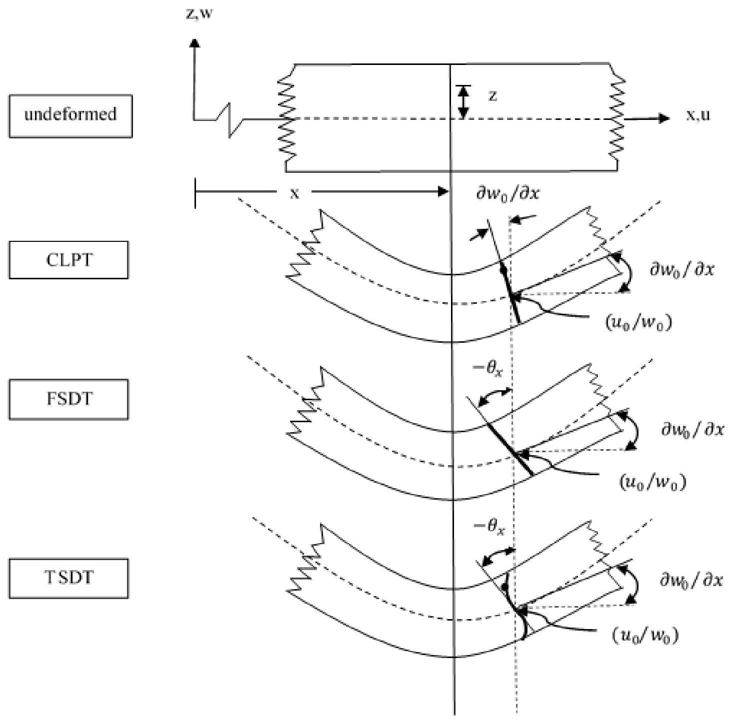


Figure 1.1 Deformation of a transverse normal according to the classical, first-order, and third-order plate theories

Higher order polynomials are used to represent displacement components through the thickness of the plate in HSDT, and the actual transverse strain/stress through the thickness. It is possible to expand the displacement field in terms of the thickness coordinate up to any desired degree. The reason for expanding the displacements up to the cubic term in the thickness coordinate is to have quadratic variation of the transverse shear strains and transverse shear stresses through each layer. This avoids the need for shear correction coefficients used in the first-order theory. However, it important to mention that the complexities in formulation and large computational effort make it economically unattractive. In addition to that the introduction of digital computer along with its capability of exponentially increasing computing speed has made the analytically difficult problems amenable through the various numerical methods and thus making the literature rich in this area. A detailed literature review on different issues have been done based on the problems are discussed in the above paragraphs. The literature review have been subdivided into two major categories such as vibration and buckling behaviour of various panel structures.

1.3 Vibration Analysis of Laminated Composite Panels

In this section the vibration behaviour of laminated structures have been discussed based on the available literature. It is well known that the vibration behaviour of laminated structures is one of the important concerns of many researchers for the prediction and design of structures using available and developed theories. Kant and Swaminathan [1] developed a mathematical model based on the higher order refined theory and solved analytically to study the free vibration behaviour of laminated composite and sandwich plates. The free vibration and stability problems of laminated (angle-ply and cross-ply) composite plates are solved using power series expansion by Matsunaga [2-3]. Putcha and Reddy [4] studied the stability and the vibration behaviour of laminated plates based on a refined plate theory. Static and vibration behaviour of laminated composite shells are studied based on the HSDT kinematic model by Reddy and Liu [5] and solved by Navier's-type exact solution. Ferreira *et al*. [6] studied buckling and vibration behaviour of isotropic and laminated plates in the framework of the FSDT. Bhar *et al*. [7] employed finite element method (FEM) to obtain the structural responses of laminated composite stiffened plates in the framework of the FSDT and the HSDT kinematics. Static and dynamic behavior of laminated composite plate is analysed by Mantari *et al*. [8] using a new higher order shear deformation theory. Free vibration behaviour of moving laminated composite plates is analysed based on the CLPT by Xiang and Kang [9]. The free vibration responses of laminated composite shells are analysed by Xiang *et al*. [10] using mesh less global collocation method in the framework of the FSDT. Bending and the vibration responses of laminated composite plates are obtained based on the FSDT mid-plane kinematics and are solved using the discrete shear gap method by Cui *et al*. [11]. Hatami *et al*. [12] studied free vibration behaviour of laminated composite plate using a mesh less local collocation method based on thin plate spline radial basis function. Viola *et al*. [13] studied the free vibration analysis of doubly-curved laminated shell panels using the Generalized Differential Quadrature (GDQ) technique in the HSDT kinematics. Tornabene *et al*. [14] analysed doubly-curved laminated composite shell panels to obtain the free vibration responses using the HSDT kinematics. Jin *et al*. [15-16] developed a unified and exact solution method based on the FSDT to study the vibration responses of various composite laminated structure including cylindrical, conical, spherical shells and annular plates. Nguyen-Van *et al*. [17] studied buckling behavior of laminated plate/shell using mixed interpolation smoothing quadrilateral element in the framework of the FSDT. Thai and Kim [18] obtained free vibration responses of laminated composite plates using two variable refined plate theories. Kumar *et al*. [19] developed a finite element (FE) model based on the higher order zigzag theory (HOZT) to obtain the structural responses of laminated composite and sandwich shells. Dozio [20] employed Ritz method to obtain the free vibration responses of single-layer and symmetrically laminated rectangular composite plates. Ngo-Cong *et al*. [21] represents free vibration responses of laminated composite plates in the framework of the FSDT using one-dimensional integrated radial basis function (IRBFN) method. Patel *et al*. [22] developed a model for free vibration behaviour of bimodular composite laminated cylindrical/conical panels in the framework of the FSDT. Khalili *et al*. [23] studied the structural responses of rectangular sandwich and laminated composite plates using a finite element based global–local theory in the framework of Galerkin. Tu *et al*. [24] developed finite element model for bending and vibration analysis of laminated and sandwich composite plates in the framework of the HSDT. Ye *et al*. [25] obtained free vibration responses of laminated composite shallow shells using the Rayleigh–Ritz method. Zhen *et al*. [26] developed an accurate higher order C^0 theory and used finite element method to obtain the free vibration responses of laminated composite and sandwich plates. Xie *et al*. [27] obtained free vibration responses of composite laminated cylindrical shells using the Haar wavelet method in the framework of the Reissner–Naghdi's shell theory. Qu *et al*. [28] employed a variational principle to obtain the vibration responses of composite laminated shells in the framework of the FSDT. The free vibration and buckling behaviour of non-homogeneous cross-ply rectangular plates are analysed based on the HSDT by Fares and Zenkour [29]. Kant and Mallikarjuna [30] obtained free vibration responses of unsymmetrically laminated multilayered plates using a refined higher order theory. Vibration and buckling responses of multi-layered composite plates are obtained by Noor [31-32].

1.4 Buckling Analysis of Laminated Composite Panels

As discussed earlier, buckling is one of the characteristic failure modes of thin structures like plates and shells so, it important to predict the critical buckling load of any structure. Buckling (thermal/mechanical) behaviour of laminated composite and sandwich plates are studied by Wu and Chen [33] using a global-higher order theory. Thermal buckling responses of anti-symmetric angle-ply laminated plate are obtained by Chang and Leu [34] based on a higher order displacement field and solved using 3-D elasticity solution. Shukla and Nath [35] computed the buckling and post-buckling load parameters of angle-ply laminated plates analytically under thermo-mechanical loading. The non- linearity in geometry matrix due to excess thermal/mechanical deformation is taken in von-Karman sense in the framework of the FSDT. Thermal buckling behaviour of laminated composite plates based on the layerwise theory investigated by Lee [36] using FEM. The buckling behaviour of cross-ply laminated orthotropic truncated conical shells are analysed using the von Karman–Donnell-type of nonlinear kinematics by Sofiyev and Kuruoglu [37]. Vosoughi *et al*. [38] presented thermal buckling and post-buckling load parameters of laminated composite beams. The temperature-dependent composite properties are taken in their analysis and the geometrical distortion is modelled using generalized Green's strain in the framework of the FSDT. Shen [39] studied the buckling responses of imperfect laminated plates in the framework of the HSDT. The buckling behaviour of composite plates is analysed by Jameel *et al*. [40] under thermal and mechanical loading. Li *et al*. [41] analysed buckling behaviour of composite stiffened laminated cylindrical shells based on a layerwise/solid-element (LW/SE) method and FEM. Buckling behaviour of laminated composite plate is studied by Fazzolari *et al*. [42] taking non- linearity in geometry matrix due to excess thermal/mechanical deformation in von-Karman sense in the framework of the HSDT. Shadmehri *et al*. [43] reported stability behaviour of conical composite shells subjected to axial compression load using linear strain–displacement relations in the framework of the FSDT. Farahmand *et al*. [44] studied thermal buckling behaviour of rectangular micro-plates using higher continuity p-version finite element method. Thermal buckling behaviour of composite laminated plates are studied by Shiau *et al*. [45] using the high-order triangular plate element in the framework of the FSDT. Kheirikhah *et al*. [46] studied bi-axial buckling behaviour of laminated composite and sandwich plates using the von-Karman kinematic non-linearity in the framework of the HSDT. Nali *et al*. [47] obtained buckling responses of laminated plates using von-Karman nonlinear kinematic in the framework of thin plate theory. Kant and Babu [48-49] developed a higher order refined finite element model to obtain the thermal buckling responses of laminated composite and sandwich plates. Panda and Singh [50] studied thermal post-buckling behaviour of laminated composite cylindrical/hyperboloidal shallow shell panel using FEM in framework of the HSDT taking the geometry nonlinearity in Green-Lagrange sense. Singh *et al*. [51] reported the buckling behaviour of laminated composite plates subjected to thermal and mechanical loading using mesh-less collocation method in the framework of the HSDT.

In addition to the above, some researcher have also analysed the buckling behaviour of laminated structures under mechanical loading using the same type of geometrical nonlinearity and in-plane mechanical loading. Grover *et al*. [52] analysed static and buckling behaviour of laminated composite and sandwich plates using an inverse hyperbolic shear deformation theory. Buckling behaviour of angle-ply composite and sandwich plates are computed by Zhen and Wanji [53]. Topal and Uzman [54] developed a model in the framework of the FSDT using four node Lagrangian finite element (FE) to investigate the buckling responses. Khdeir and Librescue [55] investigated the buckling and the free vibration responses of symmetric cross-ply laminated elastic plates based on the HSDT mid-plane kinematics. Fazzolari *et al*. [56] studied stability behaviour of composite plates in the framework of the HSDT and the geometric nonlinearity is taken in von-Karman sense. The buckling behaviour of symmetric angle-ply and cross-ply stepped flat composite columns are analysed using FEM by Akbulut *et al.* [57]. Matsunaga [58] analysed vibration and stability behaviour of cross-ply laminated shallow shells using power series expansion. Ferreira *et al*. [59] studied buckling behaviour of laminated composite plates using radial basis functions in the framework of the HSDT. Lal *et al*. [60] studied stability behaviour of laminated composite cylindrical shell panel subjected to hygro-thermo-mechanical loading taking the geometrical nonlinearity in von-Karman sense in the framework of the HSDT. Amadio and Bedon [61-62] developed an analytical model based on the Newmark's theory to obtain the load bearing capacity of laminated glass beams in out-of-plane bending and in-plane compression or shear. Chirica and Beznea [63] studied buckling behaviour of the multiple delaminated composite plates under shear and axial compression. The buckling behaviour of cross-ply laminated conical shell panels subjected to axial compression is studied by Abediokhchi *et al*. [64] in the framework of the classical shell theory. Komur *et al*. [65] obtained buckling responses of laminated composite plates with an elliptical/circular cut-out using FEM and governing equations are solved using Newton–Raphson method. Buckling and post-buckling behaviour of laminated composite plate is studied by Dash and Singh [66] using Green-Lagrange type nonlinearity in the framework of the HSDT. Seifi *et al*. [67] reported buckling responses of composite annular plates under uniform internal and external radial edge loads in the framework of the CLPT. Khalili *et al*. [68] obtained buckling load parameters of laminated rectangular plate on Pasternak foundation using the Lindstedt–Poincare perturbation technique. Cagdas [69] obtained buckling responses of cross-ply laminated shells of revolution using a curved axisymmetric shell finite element. Tang and Wang [70] studied buckling behaviour of symmetrically laminated rectangular plates with in-plane compressive loadings using the Rayleigh–Ritz method in the framework of the CLPT. Pandit *et al*. [71]

reported buckling behaviour of laminated sandwich plates based on an improved higher order zigzag theory.

1.5 Introduction of Finite Element Method and ANSYS

In this advance world, the finite element method (FEM) is widely adopted and being used as the most trustworthy tool for designing of any structure because of the precision of this method compare to other analytical or numerical methods. It plays an important role in predicting the responses of various products, parts, assemblies and subassemblies. Nowadays, FEM is extensively used in all advanced industries which saves the huge time of prototyping with reducing the cost due to physical test and increases the innovation at a faster and more accurate way. There are many optimized finite element analysis (FEA) tools are available in the market and ANSYS is one of them which is acceptable by many industries and analysts.

Nowadays, ANSYS is being used in a different engineering fields such as power generation, electronic devices, transportation, and household appliances as well as to analyse the vehicle simulation and in aerospace industries. ANSYS gradually entered into a number of fields making it convenient for fatigue analysis, nuclear power plant and medical applications. Thermal/mechanical/thermo-mechanical analysis of various structures based on the thermal and/or mechanical loading can be done with ease. ANSYS is also very useful in electro thermal analysis of switching elements of a super conductor, ion projection lithography, detuning of an HF oscillator by the mechanical vibration of an acoustic sounder.

In the present work, the vibration and buckling (thermal/mechanical) analysis is done by taking shell element SHELL281 from the ANSYS library shown in Figure 1.2 [75]. It is an eight-noded linear shell element with six degrees of freedom at each node which translation in x, y, z direction and rotation about x, y, z axis. It is well-suited for thin to moderately thick shell structures and linear, large rotation, and/or large strain nonlinear applications.

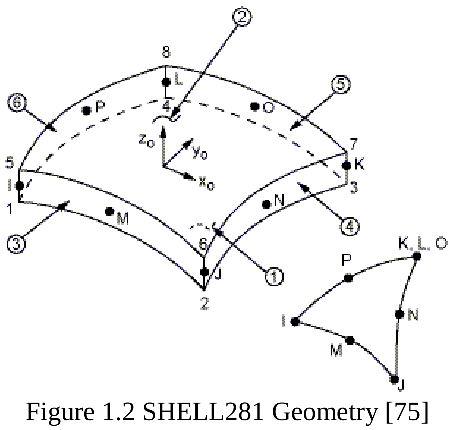


Figure 1.2 SHELL281 Geometry [75]

x_o = Element x-axis if element orientation is not provided.

x = Element x-axis if element orientation is provided.

1.6 Motivation of the Present Work

The laminated composite shell panels are of great attention to the designers because of efficient lightweight structures, due to their tailored properties as mentioned earlier. Increased uses of the composite structures especially in aeronautical/aerospace engineering have created the requirement of their analysis. These structural components are subjected to various types of combined loading and exposed to elevated thermal/mechanical/thermo-mechanical environment during their service, which often changes the original geometry of the shell panel. The changes in panel geometry and the interaction with loading condition affect the structural vibration and buckling responses greatly.

In order to achieve the light weight structures for stringent demand of weight reduction in the advanced engineering structures to conserve energy the laminated composites consisting of multiple layers are extensively employed and their usage will continue to grow as structural members. It is also important to mention that, these laminated composites are weak in shear and highly flexible in nature as compared to any other metallic plate/shell. To obtain the accurate prediction of responses of laminated composites, it is necessary and essential requirement that the displacement model must be capable to take care of the effect of shear deformation. In this regard a higher order shear deformation theory is most desirable. As discussed earlier, the structural components such as flat/curved laminated shell panels are often subjected to intense thermal/mechanical/thermo-mechanical loading and/or large amplitude vibration during their service. The geometry of the shell panel alter and stiffness matrix associated with the material are no more linear due to excess deformations and this effects has to be appropriately considered in the analysis. The vibration and buckling of structures have been received a considerable attention not only due to their wide range application, but also the challenging problems with interesting behaviour. In most of the literature, the geometry matrix associated in buckling is modelled taking into account for the non-linearity in the von-Karman sense. But the nonlinearity in von-Karman sense may not be appropriate enough for the realistic prediction of their responses. It is noted that the studies related buckling behaviour of laminated panel structure under thermo-mechanical loading need to be exploited more by using a better mathematical model for the prediction of exact behaviour of laminated structures. In addition to that the comprehensive testing of the desired responses using commercial finite element package will be a really add on to the present study.

1.7 Objectives and Scope of the Present Thesis

This thesis aims to develop a general mathematical model for laminated composite curved panel under uniform temperature based on the HSDT displacement field model. The Green-Lagrange type of strain displacement relations are employed to take care the geometrical distortion. A suitable finite element model is proposed and implemented for the discretisation of the panel model. It also aims to obtain the effect of different types panel geometries (cylindrical, spherical, elliptical, hyperboloid and flat) and other geometrical parameters (aspect ratios, thickness ratios, curvature ratios, modular ratios, support conditions and lamination schemes) on the free vibration and buckling (thermal, mechanical and thermo-mechanical) responses of the laminated composites. A detailed scope of the present study is given below:

- As a first step, free vibration behavior of laminated composite panels of various geometries (cylindrical, spherical, elliptical, hyperboloid and flat plate) has been studied using the developed mathematical model.
- The model is extended, to study the buckling temperature of laminated curved composite panels subjected to uniform temperature field through thickness by taking the geometric matrix in Green-Lagrange sense.
- The model is extended, to study the mechanical buckling behavior of laminated composite shells subjected to uniaxial and/or bi-axial loading.
- The vibration and buckling (thermal/mechanical) behaviour of laminated panels has been validated using the APDL code in ANSYS 13.0 environment.
- Finally, the parametric study of laminated composite panel have been carried out using finite element analysis software ANSYS and developed HSDT model.

Few numerical examples of laminated composite shell panel are solved using the proposed model**.** In order to check the efficacy of the present developed mathematical model and simulation model as well the convergence behaviour with mesh refinement have been computed for all possible geometries of shell panels. In addition to that, the present results are also compared with those available published literature to show the accuracy of the present developed mathematical model. Finally, a good volume of new results are computed for the future references in this field of study.

1.8 Organisation of the Thesis

The overview and motivation of the present work followed by the objectives and scope of the present thesis are discussed in this chapter. The background and state of the art of the present problem by various investigators related to the scope of the present area of interest are addresses in this chapter. This chapter divided into five different sections, the first section, a basic introduction about problem and theories used in past. In the section two, some important contributions for vibration and buckling (thermal/mechanical/thermo-mechanical) behaviour of laminated composite structures are discussed. In the section three, a brief introduction of finite element method and finite element analysis software ANSYS is presented. The motivation of the present work is discussed in fourth and in fifth objective and scope of present work is incorporated. Some critical observations are discussed in the final section. The remaining part of the thesis are organised in the following fashion.

In Chapter 2, a general mathematical formulation for the thermo-mechanical buckling and free vibration of laminated composite panel, by modelling in the framework of the HSDT under the uniform temperature distribution. The Green-Lagrange type strain displacement relations are considered to take into account the geometrical nonlinearity arising in the shell panel due to excess deformation. The steps of various energy calculations, governing equation and solution steps are discussed. Subsequently, the boundary condition and computational investigation are discussed.

Chapter 3, illustrates, the free vibration responses of laminated composite panels for various panel geometries such as cylindrical, spherical, elliptical, hyperboloid and flat panel are discussed. Detailed parametric studies of material and geometrical parameters are also discussed.

Enhancement of buckling (thermal/mechanical/thermo-mechanical) of laminated composite panels for different panel geometries and the influence of geometrical and material properties on the panel responses are discussed in Chapter 4.

Chapter 5 summarizes the whole work and it contains the concluding remarks drawn from the present study and the future scope of the work of the present study.

Some important books and publications referred during the present study have been listed in the References section.

In order to achieve the objective and scope of the present work discussed above in this chapter, there is need to know the state of art of the problem for that a detailed review of earlier work done in the same field have been discussed thoroughly in the next chapter.

The following papers are prepared based on the work presented in the thesis.

In International Journals:

1. P. V. Katariya and S. K. Panda, "Vibration and Thermal Buckling Analysis of Curved Panels," Aircraft Engineering and Aerospace Technology. (Revised)

2. P. V. Katariya and S. K. Panda, "Stability and Free Vibration Behaviour of Laminated Composite Plate under Thermo-mechanical Loading," International Journal of Applied and Computational Mathematics. (communicated)

In Conference Proceedings:

3. P. V. Katariya and S. K. Panda, "Modal Analysis of Laminated Composite Spherical Shell Panels using Finite Element Method," Proceedings of International Conference on Structural Engineering and Mechanics (ICSEM), Dec 20-22 2013, NIT Rourkela, Odisha, India.

4. P. V. Katariya and S. K. Panda, "Stability and vibration behavior of laminated composite and sandwich structures," Proceedings of 2nd KIIT International Symposium on Advances in Automotive Technology (KIIT-SAAT), Dec 20-21 2013, KIIT Bhubaneswar, Odisha, India.

5. P. V. Katariya and S. K. Panda, "Thermo-Mechanical Stability Analysis of Composite Cylindrical Panels," ASME 2013 Gas Turbine India conference (ASME GTINDIA 2013), Dec 5-6 2013, Bangalore, Karnataka, India.

6. P. V. Katariya and S. K. Panda, "A parametric study on free vibration behavior of laminated composite curved panels," Proceedings of International Conference on Advances in Mechanical Engineering (ICAME), May 29-31 2013, COEP, Pune, Maharashtra, India.

7. P. V. Katariya and S. K. Panda, "Stability Analysis of Laminated Composite Cylindrical Shell Structure under Uniaxial loading," Proceedings of All India Seminar on Recent Advances in Mechanical Engineering, Mar 16-17 2013, Bhubaneswar, Odisha, India.

8. P. V. Katariya and S. K. Panda, "Thermo-mechanical stability analysis of laminated composite plate using FEM," Proceedings of 1st KIIT International Symposium on Advances in Automotive Technology (KIIT-SAAT), Jan 11-12, 2013, KIIT Bhubaneswar, Odisha, India.

CHAPTER 2

GENERAL MATHEMATICAL FORMULATION

2.1 Introduction

In today's modern world, many weight sensitive industries are using high performance laminated composite structures and their components such as laminated curved/flat panels are need to be designed and analysed very carefully. These structural components are very often subjected to vibration and buckling (thermal/mechanical/thermo-mechanical) due to excess deformation during their service life. Hence, these components must be reliable enough and should have good load bearing capacity and the demands for detailed and realistic studies of different structural responses have increase very significantly.

It is important to mention that the original geometry of the panel is distorted because of the excess deformation and induced stress and if the deformation is large, the basic geometry of the panel changes which affects the stiffness properties of the structure largely. As earlier mentioned the geometric strain associated with buckling phenomena is nonlinear in nature. It is well known that the total deformation occurred in a material continuum is the sum of translation, rotation and distortion components. If the structural elements undergoes severe nonlinearity then not only the distortion component but also the other two components in deformation play important role to derive the actual strain-displacement relation. The laminated composite structural components are extremely flexible as compared to other metallic components so the large deformation terms and the higher order shear deformation terms arises during the mathematical modelling are more important for an accurate prediction of the frequency and the buckling load parameter. A thorough literature review in the previous chapter clearly shows that various studies have been done previously on different types of problem. But very few studies have been reported the buckling behaviour by taking the geometrical nonlinearity in Green-Lagrange sense in the framework of the HSDT and taking all the higher order term in the mathematical formulation for laminated composite shell panel. In addition to this, the detailed study has been validated with the commercially available finite element (FE) software package ANSYS which are less in number.

In this chapter a general mathematical formulation of doubly curved shell panel is developed on the basis of basic assumptions. The system of governing equations for the vibration and buckling (thermal/mechanical/thermo-mechanical) characteristics of laminated composite panels derived using the variational approach. A finite element model is employed to discretise the present panel model through the chosen displacement field. For the buckling analysis of laminated composite panels, the geometric nonlinearity has been considered in Green-Lagrange strain-displacement type for the formulation and discussed in this chapter.

2.2 Assumptions

The mathematical formulation is based on the following assumptions:

- The basic geometric configuration of the problem considered here is a doubly curved shell panel on a rectangular plan form.
- The doubly curved panel geometry has been chosen as a basic configuration, so that depending on the values of curvature parameters, flat, cylindrical, spherical, hyperboloid and elliptical panel configuration can be considered.
- The middle plane of the doubly curved panel is taken as the reference plane.
- A two dimensional approach has been adopted to model a three dimensional behaviour of shell.
- Perfect bond exists between fibres and matrices so that no slippage occurs at the interface.
- A uniform temperature field is considered for the present analysis through the thickness.
- The composite material properties are assumed to be temperature independent.
- The laminated panel consists of number of layers bonded together where each layer is treated as homogeneous and orthotropic.

2.3 Displacement Field and Geometry of the Shell

The doubly curved panel is classified according to its curvature parameter, such as cylindrical (where one curvature is infinite), spherical (where both curvature are same), flat panel (where both curvatures are infinite) and doubly curved (where the two curvatures are different). In this work, a panel is assumed, which is composed of *N* number of anisotropic layers of uniform thickness *h*, length *a*, and width *b* which is shown in Figure 2.1. The values of the principal radii of the curvature are denoted by R_x and R_y. The following displacement field for the laminated shell panel based on the HSDT proposed by Dash and Singh [66] is considered to derive the mathematical model.

$$\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) + z^2\phi_x(x,y,t) + z^3\psi_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) + z^2\phi_y(x,y,t) + z^3\psi_y(x,y,t) \\
w(x,y,z,t) &= w_0(x,y,t) + z\theta_z(x,y,t)
\end{aligned} \tag{2.1}$$

where, *u*, *v* and *w* denote the displacements of any point along the (*x*, *y*, *z*) coordinates and *t* is the time. u_0, v_0 and w_0 are the in-plane and transverse displacements of a point (*x*, *y*) on the mid-plane, respectively and θ_x, θ_y and θ_z are the rotations of normal to mid-plane. The parameters ϕ_x, ϕ_y, ψ_x and ψ_y are the corresponding higher order deformation terms in the Taylor series expansion and are also defined at the mid-plane.

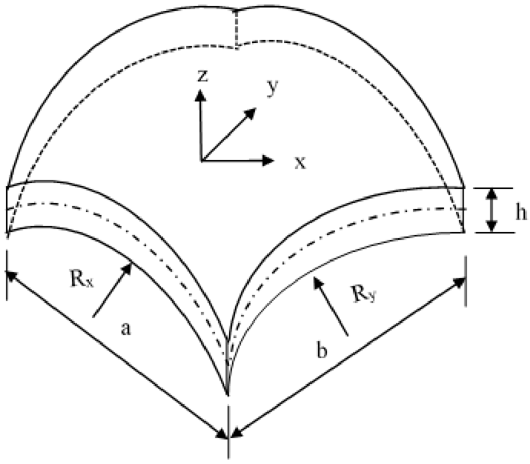


Figure 2.1 Laminated composite doubly curved shell geometry

2.4 Strain-Displacement Relation

The strain displacement relations for any material continuum are adopted for the laminated shell panel which are expressed as Reddy [73]:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{\partial v}{\partial y}\\ \dfrac{\partial w}{\partial z}\\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\\ \dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\\ \dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\end{Bmatrix}+\begin{Bmatrix}\dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial v}{\partial x}\right)^2+\left(\dfrac{\partial w}{\partial x}\right)^2\right]\\ \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial y}\right)^2+\left(\dfrac{\partial v}{\partial y}\right)^2+\left(\dfrac{\partial w}{\partial y}\right)^2\right]\\ \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial z}\right)^2+\left(\dfrac{\partial v}{\partial z}\right)^2+\left(\dfrac{\partial w}{\partial z}\right)^2\right]\\ \left[\left(\dfrac{\partial u}{\partial x}\right)\left(\dfrac{\partial u}{\partial y}\right)+\left(\dfrac{\partial v}{\partial x}\right)\left(\dfrac{\partial v}{\partial y}\right)+\left(\dfrac{\partial w}{\partial x}\right)\left(\dfrac{\partial w}{\partial y}\right)\right]\\ \left[\left(\dfrac{\partial u}{\partial x}\right)\left(\dfrac{\partial u}{\partial z}\right)+\left(\dfrac{\partial v}{\partial x}\right)\left(\dfrac{\partial v}{\partial z}\right)+\left(\dfrac{\partial w}{\partial x}\right)\left(\dfrac{\partial w}{\partial z}\right)\right]\\ \left[\left(\dfrac{\partial u}{\partial y}\right)\left(\dfrac{\partial u}{\partial z}\right)+\left(\dfrac{\partial v}{\partial y}\right)\left(\dfrac{\partial v}{\partial z}\right)+\left(\dfrac{\partial w}{\partial y}\right)\left(\dfrac{\partial w}{\partial z}\right)\right]\end{Bmatrix}$$

$$or, \{\varepsilon\}=\{\varepsilon_l\}+\{\varepsilon_{nl}\} \tag{2.2a}$$

where, $\{\varepsilon_l\}$ and $\{\varepsilon_{nl}\}$ are the linear and nonlinear mid-plane strain vectors, respectively. It is important to mention that the nonlinear terms associated with buckling behaviour are taken as,

$$\left\{\varepsilon_{nl}\right\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]\\ \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\\ \left[\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)+\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right]\end{Bmatrix} \tag{2.2b}$$

Substituting Eq. (2.1) into Eq. (2.2a) the strain displacement relations of the laminated shell panel are further expressed as:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \varepsilon_z^0\\ \varepsilon_{xy}^0\\ \varepsilon_{xz}^0\\ \varepsilon_{yz}^0\end{Bmatrix}+\begin{Bmatrix}\varepsilon_x^4\\ \varepsilon_y^4\\ \varepsilon_z^4\\ \varepsilon_{xy}^4\\ \varepsilon_{xz}^4\\ \varepsilon_{yz}^4\end{Bmatrix}+z\left[\begin{Bmatrix}k_x^1\\ k_y^1\\ k_z^1\\ k_{xy}^1\\ k_{xz}^1\\ k_{yz}^1\end{Bmatrix}+\begin{Bmatrix}k_x^5\\ k_y^5\\ k_z^5\\ k_{xy}^5\\ k_{xz}^5\\ k_{yz}^5\end{Bmatrix}\right]+z^2\left[\begin{Bmatrix}k_x^2\\ k_y^2\\ k_z^2\\ k_{xy}^2\\ k_{xz}^2\\ k_{yz}^2\end{Bmatrix}+\begin{Bmatrix}k_x^6\\ k_y^6\\ k_z^6\\ k_{xy}^6\\ k_{xz}^6\\ k_{yz}^6\end{Bmatrix}\right]+$$

$$z^3\left[\begin{Bmatrix}k_x^3\\ k_y^3\\ k_z^3\\ k_{xy}^3\\ k_{xz}^3\\ k_{yz}^3\end{Bmatrix}+\begin{Bmatrix}k_x^7\\ k_y^7\\ k_z^7\\ k_{xy}^7\\ k_{xz}^7\\ k_{yz}^7\end{Bmatrix}\right]+z^4\begin{Bmatrix}k_x^8\\ k_y^8\\ k_z^8\\ k_{xy}^8\\ k_{xz}^8\\ k_{yz}^8\end{Bmatrix}+z^5\begin{Bmatrix}k_x^9\\ k_y^9\\ k_z^9\\ k_{xy}^9\\ k_{xz}^9\\ k_{yz}^9\end{Bmatrix}+z^6\begin{Bmatrix}k_x^{10}\\ k_y^{10}\\ k_z^{10}\\ k_{xy}^{10}\\ k_{xz}^{10}\\ k_{yz}^{10}\end{Bmatrix} \tag{2.3}$$

Now the above strain displacement relations Eq. (2.3) can be rearranged as

$$\{\varepsilon\}=[T_l]\{\bar{\varepsilon}_l\}+[T_{nl}]\{\bar{\varepsilon}_{nl}\} \tag{2.4}$$

where, $\{\bar{\varepsilon}_l\}$ and $\{\bar{\varepsilon}_{nl}\}$ are the linear and nonlinear mid-plane strain vectors. $[T^l]$ and $[T^{nl}]$ are the linear and nonlinear thickness coordinate matrix.

The mid plane linear strain vector is given by

$$\{\bar{\varepsilon}_l\}=\left[\varepsilon_x{}^0\ \varepsilon_y{}^0\ \varepsilon_{xy}{}^0\ \varepsilon_{xz}{}^0\ \varepsilon_{yz}{}^0\ \ k_x{}^1\ k_y{}^1\ k_{xy}{}^1\ k_{xz}{}^1\ k_{yz}{}^1\ k_x{}^2\ k_y{}^2\ k_{xy}{}^2\ k_{xz}{}^2\ k_{yz}{}^2 k_x{}^3\ k_y{}^3\ k_{xy}{}^3\ k_{xz}{}^3\ k_{yz}{}^3\right]^T$$

The mid plane nonlinear strain vector is given by

$$\{\bar{\varepsilon}_{nl}\} = \left[\varepsilon_x{}^4\ \varepsilon_y{}^4\ \varepsilon_{xy}{}^4\ \ k_x{}^5\ k_y{}^5\ k_{xy}{}^5\ \ k_x{}^6\ k_y{}^6\ k_{xy}{}^6\ k_x{}^7\ k_y{}^7\ k_{xy}{}^7\ k_x{}^8\ k_y{}^8\ k_{xy}{}^8\ \ k_x{}^9\ k_y{}^9\ k_{xy}{}^9\ k_x{}^{10}\ k_y{}^{10}\ k_{xy}{}^{10}\ \right]^T$$

Here the terms contained in $\{\bar{\varepsilon}_l\}$ and $\{\bar{\varepsilon}_{nl}\}$ having superscripts 0, 1, 2, 3 and 4, 5, 6, 7, 8, 9, 10 are the bending, curvature and higher order terms. $[T]$ is the function of thickness coordinate. The thickness coordinate matrix and the nonlinear strain terms are provided in Appendix A.

2.5 Constitutive Relation

The stress-strain relations for k^{th} orthotropic lamina of composite matrix in material coordinate axes subjected to uniform temperature field are expressed as Reddy [73]:

$$\{\sigma\}^k = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^k = \left[\bar{C}\right]^k \{\varepsilon\}^k = \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{13}} & \overline{C_{14}} & \overline{C_{15}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{23}} & \overline{C_{24}} & \overline{C_{25}} & \overline{C_{26}} \\ \overline{C_{13}} & \bar{C}_{23} & \bar{C}_{33} & \bar{C}_{34} & \overline{C_{35}} & \overline{C_{36}} \\ \overline{C_{14}} & \bar{C}_{24} & \bar{C}_{34} & \bar{C}_{44} & \overline{C_{45}} & \overline{C_{46}} \\ \overline{C_{15}} & \overline{C_{25}} & \overline{C_{35}} & \overline{C_{45}} & \overline{C_{55}} & \overline{C_{56}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{36}} & \overline{C_{46}} & \bar{C}_{56} & \overline{C_{66}} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^k \tag{2.5}$$

where, $\{\sigma\}^k$ is the total stress vector measured at the stress free state at T_{ref}, $\{\varepsilon\}^k$ is the strain vector for the k^{th} layer and the elements of the stiffness coefficients matrix, $\bar{C}_{ij}{}^k$ $(i, j = 1, \ldots, 6)$ can be obtained using the appropriate transformation on the stiffness matrix $[C_k]$ corresponding to material principal directions.

Thermo-mechanical in-plane generated forces can be obtained by integrating the Eq. (2.5) over the thickness of the shell panel and can be expressed in matrix form as follows

$$\begin{Bmatrix} N \\ M \\ P \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left(\left[\bar{Q}\right]^k + \left[\bar{Q}\right]^k \{\alpha\}^k \Delta T\right)\left(1, z, z^2\right) dz \tag{2.6}$$

where, $\{N\}, \{M\}$ and $\{P\}$ are the resultant vectors of compressive in-plane forces, moments and the higher order terms due to thermal and mechanical load applied individual or combined action of forces on composite matrix. $\left[\overline{Q}\right]^k$ is transferred reduced stiffness matrix of lamina for any k^{th} lamina and $\{\alpha\}^k = \{\alpha_1 \quad \alpha_2 \quad \alpha_{12}\}$ is the co-efficient of thermal expansion in the respective direction. .

2.6 Energy Calculation

As a first step, the global displacement vector can be expressed in matrix form

$$\{\overline{\delta}\} = \begin{Bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{Bmatrix} = [f]\{\delta\} \tag{2.7}$$

where, $[f]$ and $\{\delta\} = \{u \quad v \quad w \quad \theta_x \quad \theta_y \quad \theta_z \quad \phi_x \quad \phi_y \quad \psi_x \quad \psi_y\}^T$ are the functions of thickness coordinate and the displacement vector at mid plane of the panel, respectively.

The kinetic energy expression (T) of a laminated composite panel can be expressed as:

$$T = \frac{1}{2}\int_V \rho \left\{\dot{\overline{\delta}}\right\}^T \left\{\dot{\overline{\delta}}\right\} dV \tag{2.8}$$

where, ρ, and $\left\{\dot{\overline{\delta}}\right\}$ are the density and the first order differential of the displacement vector with respect to time, respectively.

Using Eq. (2.7) the kinetic energy Eq. (2.8) for 'N' number of orthotropic layered composite panel can be written as

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{\dot{\delta}\right\}^T [f]^T \rho^k [f] \left\{\dot{\delta}\right\} dz \right) dA = \frac{1}{2}\int_A \left\{\dot{\delta}\right\}^T [m] \left\{\dot{\delta}\right\} dA \tag{2.9}$$

where, $[m] = \sum_{k=1}^{N} \int_{z_{k=1}}^{z_k} \left([f]^T \rho^k [f]\right) dz$ is the inertia matrix.

The strain energy ($U_{\text{S.E.}}$) of a laminated composite panel can be expressed as:

$$U_{S.E.} = \frac{1}{2}\int_V \{\varepsilon\}_i^T \{\sigma_i\} dV \tag{2.10}$$

By substituting the strains and the stresses as given in Eqs. (2.4) and (2.5) into Eq. (2.7) the strain energy expression can be expressed as:

$$U_{S.E.} = \frac{1}{2}\int_V \left\{\varepsilon^T\right\}^k \left(\left[\overline{Q}\right]^k \{\varepsilon\}^k - \left(\left[\overline{Q}\right]\{\alpha\}\right)^k \Delta T\right) dV \tag{2.11}$$

The work done W due to the generated in-plane compressive thermal and/or mechanical force resultants N in Green–Lagrange sense for the curved panel can be obtained in similar fashion as in Eq. (2.4)

$$W = \int [\bar{\varepsilon}_{nl}]^T \{N\} dV \tag{2.12}$$

where, $\{N\} = \{N_x \quad N_y \quad N_{xy}\}^T$ is the thermal and/or mechanical load vector and $\left[\bar{\varepsilon}_{nl}\right]$ is the geometric strain vector.

The above expression as given in Eq. (2.12) is linearized following the procedure as adopted in Panda [74] and conceded to

$$W = \frac{1}{2}\int_V \left\{\bar{\varepsilon}_{nl}\right\}^T [D_G] \left\{\bar{\varepsilon}_{nl}\right\} dV \tag{2.13}$$

The values of the linearized geometric strain vector $\left[\bar{\varepsilon}_{nl}\right]$ and the material property matrix $[D_G]$ are presented in the Appendix B.

2.7 Finite Element Formulation

In this present work, a nine noded isoparametric quadrilateral Lagrangian element having 90 degrees of freedom (DOFs) per element is employed. The details of the element can be seen in Cook *et al*. [72].

The global displacement vector $(\delta) = \left[u \quad v \quad w \quad \theta_x \quad \theta_y \quad \theta_z \quad \phi_x \quad \phi_y \quad \psi_x \quad \psi_y\right]$ can be presented to the form by employing FEM

$$\{\delta\} = [N_i]\{\delta_i\} \tag{2.14}$$

where, $[N_i]$ and $\{\delta_i\}$ are the nodal interpolation function and displacement vector for i^{th} node, respectively.

By substituting of Eq. (2.14) into Eqs. (2.9), (2.11) and (2.13) the kinetic energy, strain energy and the work done expressions can be further expressed as

$$T = \int_A \left(\left[N_i \right]^T \left[m \right] \left[N_i \right] dA \right) \left\{ \ddot{\delta} \right\} \tag{2.15}$$

$$U_{S.E.} = \frac{1}{2} \int_A \left(\left\{ \delta \right\}_i^T \left[B \right]_i^T \left[D \right] \left[B \right]_i \left\{ \delta \right\}_i dA \right) - \left\{ F \right\}_i \tag{2.16}$$

$$W = \frac{1}{2} \int_A \left(\left\{ \delta \right\}_i^T \left[B_{nl} \right]_i^T \left[D_G \right] \left[B_{nl} \right]_i \left\{ \delta \right\}_i dA \right) \tag{2.17}$$

where, $\left\{ \varepsilon_l \right\}_i = \left[B_l \right]_i \left\{ \delta \right\}_i, \left\{ \varepsilon_{nl} \right\}_i = \frac{1}{2} \left[B_{nl} \right]_i \left\{ \delta \right\}_i = \frac{1}{2} \left[A(\delta) \right]_i \left[G \right]_i \left\{ \delta \right\}_i, \left\{ F \right\}_i = \int_A \left[B_l \right]_i^T \left\{ N \right\} dA$,

$$\left[D \right] = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left[T \right]^T \left[\overline{Q} \right] \left[T \right] dz$$

$\left[B_l \right]$ and $\left[G \right]$ are the product form of the differential operator and nodal interpolation function in the linear strain terms and geometric strain terms, respectively. The expression of [G] arising due to the Green-Lagrange nonlinearity in the nonlinear stiffness matrices is provided in Appendix C.

2.8 Governing Equation

The final form of governing differential equation of composite panels can be obtained using Hamilton's principle. This result in

$$\delta \int_{t_1}^{t_2} L dt = 0 \tag{2.18}$$

where, $L = T - \left(U_{S.E.} + W \right)$

2.9 Solution Technique

To analyse the free vibration and buckling (thermal/mechanical/thermo-mechanical) behaviour of laminated composite panel, the governing equations are deduced from the Eq. (2.18) and can be presented as follows:

$$\left\{K_S - \omega^2 [M]\right\}\{\delta\} = 0 \tag{2.19}$$

$$\left(K_S - \lambda_{cr} K_G\right)\{\delta\} = 0 \tag{2.20}$$

where, $[M] = \int_{dA} [N_i]^T [m][N_i] dA$ and $[K_G] = \int_{dA} [B_{nl}]_i^T [D_G][B_{nl}]_i dA$. λ_{cr}, $\{\delta\}$, $[K_S]$ and $[K_G]$ are the critical buckling load parameter, the displacement vector, the stiffness matrix and global geometric stiffness matrix, respectively.

The matrix equations have been solved numerically using the FEM steps and the detailed steps are presented in Figure 2.2.

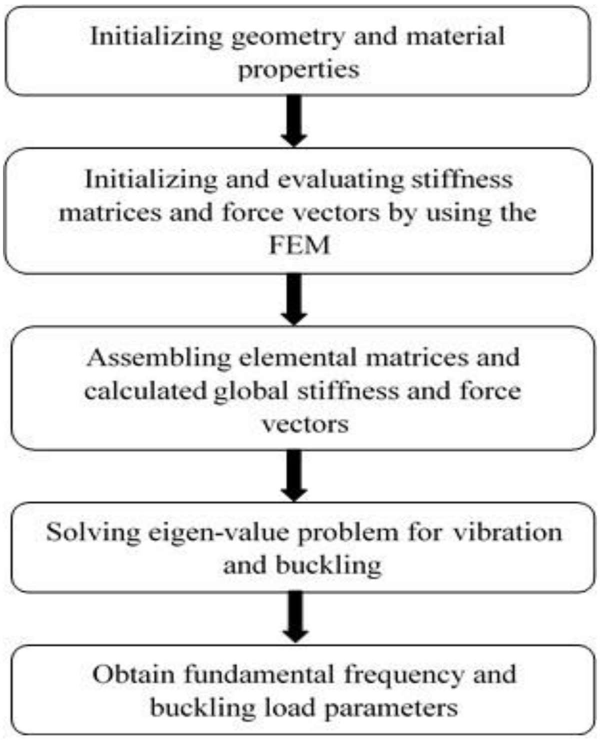


Figure 2.2 Representation of the solutions steps

2.10 Support Conditions

The main purpose of the support condition is to avoid rigid body motion as well as to decrease the number of unknowns of a system for ease in calculation and also the singularity in the matrix equation can be avoided. In this work, kinematical constraint conditions are applied as the model is developed using the displacement based finite element i.e., all the unknowns are defined in terms of displacement only.

The following sets of boundary conditions are used for the present work. However, the mathematical formulation, which is general in nature, does not put any limitations.

a) Simply supported boundary condition (S):

$v_0 = w_0 = \theta_x = \theta_z = \phi_x = \psi_x = 0$ at $x = 0, a$

$u_0 = w_0 = \theta_y = \theta_z = \phi_y = \psi_y = 0$ at $y = 0, b$

b) Clamped boundary condition (C):

$u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = \phi_x = \phi_y = \psi_x = \psi_y = 0$ at $x = 0, a$ and $y = 0, b$

2.11 Computational Investigations

For the computational purpose, a computer code has been developed in MATLAB 7.10 environment for the vibration and buckling (thermal/mechanical/thermo-mechanical) analysis of laminated composite panel. Although, the present code can be done using other languages such as FORTRAN and/or C++ but MATLAB is more user friendly and it is easy to implement. The MATLAB suffers from two major problems namely: inadequate memory space and computational time is more in comparison to other two types as discussed earlier. Due to the revolutionary change in digital computers, an Intel (R) Core (TM) i5-2400 CPU @ 3.10 GHz, 3.10 GHz and 4 GB RAM system is used for the present numerical analysis. However, it can be further improved by using an Intel (R) Core (TM) i7 which is also available in market. The main advantages of MATLAB are that, the error accumulations due to the numerical analysis and selection of methodology are taken care by MATLAB to get the desired output by selecting the appropriates syntax from its given library. The code has been developed in such a way that it can be compute the different types of problems of laminated composite panels. The developed code has been employed to solve the free vibration and buckling (thermal/mechanical/thermo-mechanical) of laminated composite panels. In addition to that the analysis has been carried out using simulation model developed using the ANSYS parametric design language code in ANSYS 13.0 environment for the laminated composite panels which is less time consuming and accepted by many industries.

2.12 Summary

The main aim of this present chapter is to develop a general mathematical model for the computer implementation of the proposed problem i.e., the free vibration and the buckling (thermal/mechanical/thermo-mechanical) analysis of laminated composite structures. The necessity and requirement of the problem and their background were discussed in the first section. A few essential assumptions were made in the Section 2.2. Then, in Section 2.3 the geometry of the shell panel and the assumed higher order displacement field were stated. In Section 2.4 the strain-displacement relations in Green-Lagrange sense and subsequent strain vectors evaluated. The mechanics of laminated composite was presented in Section 2.5. The general thermo-elastic constitutive relations for laminated composite and the resultant in-plane thermal/mechanical forces were discussed in Section 2.6. Then, in Section 2.7 various energies and the work done due to the in-plane thermal and/or mechanical load were calculated. The mathematical model for the proposed panel problem was discretised with the help of finite element in Section 2.8. Then, in Section 2.9 the governing equation of motion for the laminated panel were obtained using Hamilton’s principle. A detail discussion on the solution technique and the necessary assumptions were presented in Section 2.10. Finally, the types of support conditions were presented in Section 2.11 for the numerical analysis. Computational investigation was discussed briefly in Section 2.12.

In the following chapters the solution of governing equation for various kinds of panel problems such as free vibration and buckling (thermal/mechanical/thermo-mechanical) of laminated composite shell panel have been investigated in details for different parameters and discussed.

CHAPTER 3

FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE SHELL PANELS

3.1 Introduction

In this chapter, the free vibration responses of laminated composite shell panel have been investigated using the present developed mathematical model as discussed in the previous chapter. The first mode frequency of any structural system is more important to know not only to avoid the resonance but also an optimal design of the structures or the structural components. It has the highest time period and affects the system responses greatly. It is very well known that the laminated structural components are under the influence of various combined loading and constrained conditions during their service life. This affects the original geometry of the structure largely and the structural components are distorted which changes the entire situation in the structural analysis.

The aim of this chapter is to state the governing equation of free vibration of laminated composite shell panel and the solution steps to obtain the desired output. A detailed parametric study and their effects on the free vibration and the fundamental frequencies are obtained for different types of shell geometries.

3.2 Governing Equation and Solution

The governing equilibrium system equations of free vibration of laminated composite shell panel are obtained initially by dropping the appropriate terms from the Eq. (2.19) and that may be expressed as:

$$\left\{K_s - \omega^2[M]\right\}\left\{\delta\right\} = 0 \tag{3.1}$$

where, $\left[K_s\right]$ is the stiffness matrix, ω is the critical frequency parameter, $\left[M\right]$ is the mass matrix and $\left\{\delta\right\}$ is the displacement vector.

The Eq. (3.1) has been solved using the steps as discussed in the Chapter 2 and the desired responses are obtained and discussed.

3.3 Results and Discussions

A finite element code is developed in MATLAB 7.10 based on the developed mathematical shell panel model as stated and detailed is discussed in Chapter 2. The free vibration analysis of laminated composite shell panels have been obtained for ten degrees of freedom (DOFs) model. The validation and accuracy of the present algorithm is examined by comparing the results with those available in the literature. In addition to this, a simulation model is also developed in ANSYS using ANSYS parametric design language (APDL) code to cross check the present mathematical model. The developed model is validated by comparing the responses obtained using the MATLAB code and ANSYS (using Block-Lanczos method) with those available published literature. It is observed from the validation and convergence study that the present results are showing good agreement with the available literatures. The effect of different combinations of parameters such as the curvature ratio (R/a), the thickness ratio (a/h), the aspect ratio (a/b), the modular ratio (E_1/E_2), the lay-up scheme and the support condition on the composite shell panel vibration responses are discussed. The non-dimensionalised fundamental frequency is taken as $(\varpi) = \omega b^2 / \sqrt{\rho/(E_2 h^2)}$, and the same is being used in the throughout analysis if it is not stated elsewhere.

3.4 Convergence and Validation Study of Free Vibration

In this section various numerical examples have been solved to check the convergence and the efficacy of the proposed developed model. The results are compared with those available published literature to validate the present developed mathematical model and ANSYS model. As a first step, the convergence behaviour of the vibration responses are presented in Figure 3.1 and Figure 3.2 for a simply supported cylindrical and flat panel, respectively. The results are compared using the material properties M1 for the cylinder and M2 for the flat panel same as Mantari *et al*. [8] and tabulated in Table 3.1.

Table 3.1 Material properties of the laminated composite structures

M1:	E_1/E_2=25 $\nu_{12}=\nu_{13}$=0.25	$G_{12}=G_{13}=0.5E_2$ ρ=1	$G_{23}=0.2E_2$
M2:	E_1/E_2=open $\nu_{12}=\nu_{13}$=0.25	$G_{12}=G_{13}=0.6E_2$ ρ=1	$G_{23}=0.5E_2$

Figure 3.1 presents the non-dimensional fundamental frequency response for three different cross-ply [($0^0/90^0$), ($0^0/90/^00^0$) and ($0^0/90^0/90^0/0^0$)] laminated composite cylindrical panel by taking same geometrical parameters (a/h=10 and R/a=10) as in Mantari *et al*. [8].

Similarly, the non-dimensional fundamental frequency responses are obtained for cross-ply ($0^0/90^0/90^0/0^0$) laminated composite flat panel using the same properties as in earlier case for two modular ratios (E_1/E_2=30 and 40) and the responses are plotted in Figure 3.2. It can be seen that the results are converging well for both the cases as with mesh refinement. It is also observed that a (6×6) mesh is sufficient for the computation of the natural frequency. Based on the convergence study a (6×6) mesh has been used to compute the new results for throughout the analysis.

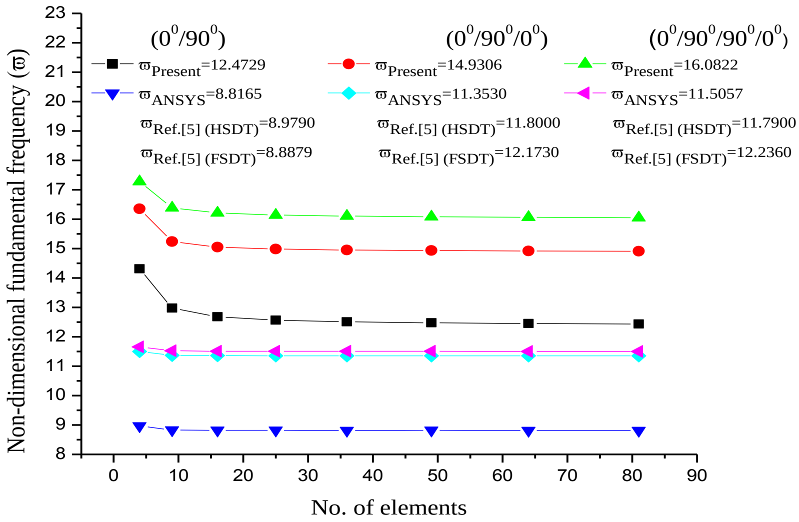


Figure 3.1 Convergence and comparison study of simply supported laminated composite cylindrical panel (R/a=10 and a/h=10)

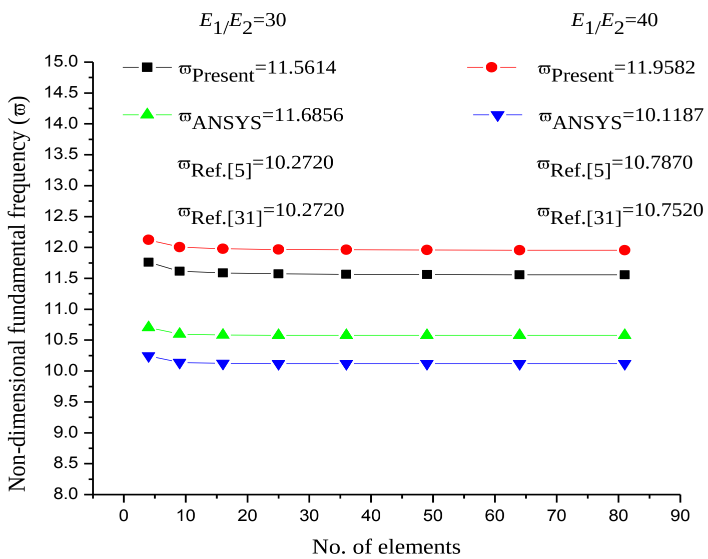


Figure 3.2 Convergence and comparison study of simply supported cross-ply ($0^0/90^0/90^0/0^0$) laminated composite flat panel (a/h=5)

In addition to the above, few more cases of convergence study have been shown for the free vibration of laminated composite cylindrical and flat panels. All the responses are obtained in ANSYS for different parameters using the said material properties as M1 and M2 for the cylindrical and the flat panel, respectively. Figure 3.3 represents the non-dimensional natural frequencies of laminated composite cylindrical panel for different lamination schemes [($0^0/90^0$), ($0^0/90^0/0^0$), ($0^0/90^0/90^0/0^0$) and ($0^0/90^0/0^0/90^0$)], thickness ratio (a/h=10) and all edges clamped support condition. It is observed from the Figure 3.3 that as the lamination scheme changes (number of layer increases) the non-dimensional fundamental frequency parameter increases. Similarly, Figure 3.4 represents the natural frequencies of square four layered anti-symmetric cross-ply $(0^0/90^0)_2$ laminated composite flat panel. The results are obtained for three different modular ratios (E_1/E_2=3, 10 and 20), two thickness ratios (a/h=5 and 100) and one edge clamped and other edges free support condition. The results clearly show that the present ANSYS model is showing well convergence. It is observed from the Figure 3.4 that the non-dimensional fundamental frequencies are increases with an increase in the modular ratios and the thickness ratios.

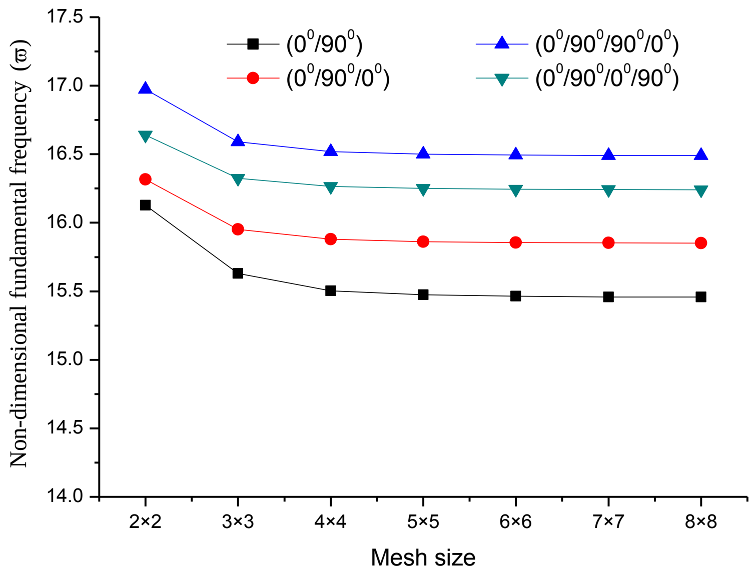


Figure 3.3 Convergence study of laminated composite cylindrical panel of different lamination schemes for all edges clamped support condition (a/h=10 and R/a=100)

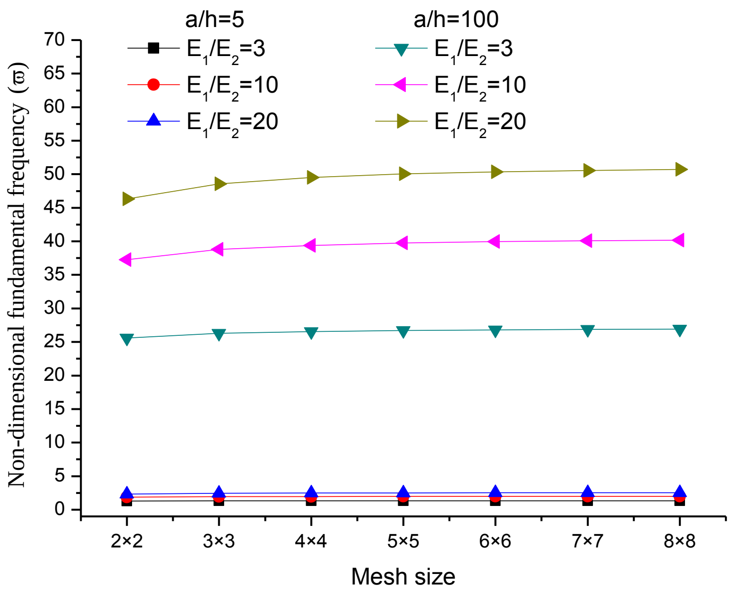


Figure 3.4 Convergence study of laminated composite flat panel of different lamination schemes and thickness ratios for one edge clamped and other edges free support condition

As discussed in the above paragraph one more case has been examined for spherical shell panel for a square simply supported (SSSS) cross-ply [($0^0/90^0$), ($0^0/90^0/0^0$) and ($0^0/90^0/90^0/0^0$)] panel using the material properties as M2. The results are computed for two thickness ratios (*a*/*h*=10 and 100) in ANSYS using APDL code and presented in Figure 3.5. It is observed from the figure that the results are showing good agreement with the available references.

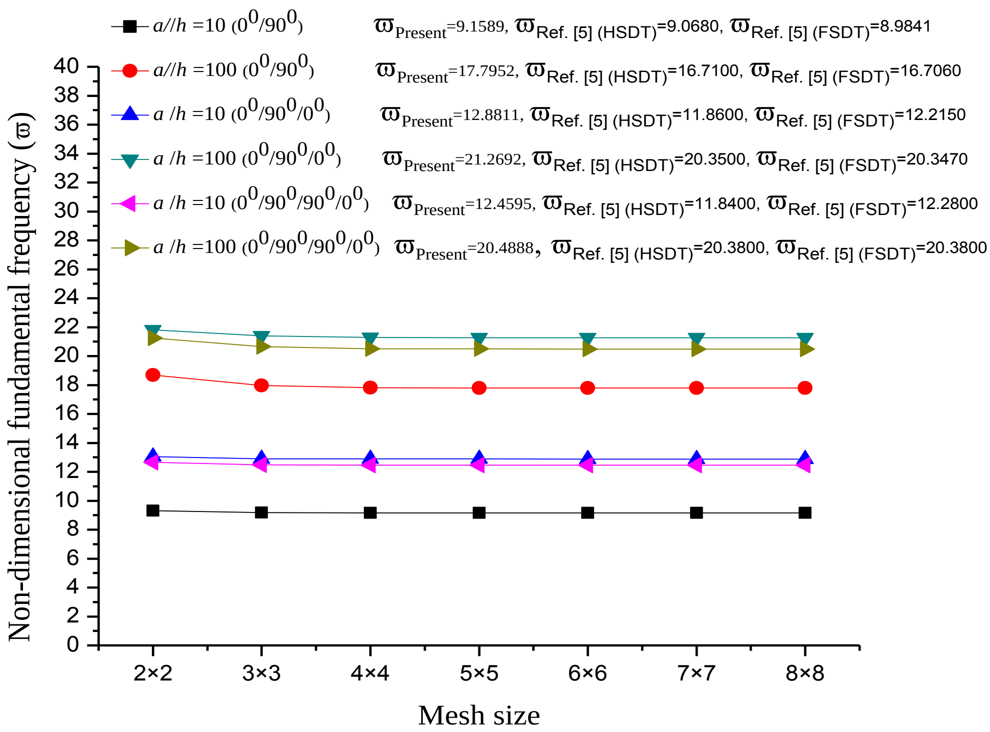


Figure 3.5 Convergence study of simply supported laminated composite spherical shell panel (E_1/E_2=25 and R/a=10)

3.5 Numerical Examples

The laminated structures are composed of various geometries of flat/curved shell panels and named accordingly to their curvature. The present numerical model is developed for any general case of shallow shell panel which has the capability to achieve different types of shell geometries. The effect of various geometric and material properties on the free vibration responses are discussed for each type of shell panel by solving different numerical examples. The numerical experimentations have been obtained for different types of curved and flat panels based on their types of curvature as discussed. The shell panels are named as spherical panel, when both of the curvature parameters are equal, i.e., $R_1=R_2=R$. Similarly, the panels are so called as flat panels (plate) if both the curvatures are infinity. If the panel is having one of the curvature is zero i.e., named as cylindrical shells and so on the hyperboloid and elliptical are classified. Finally, the vibration behaviour of these structural panels have been analysed and discussed their importance. The effect of different parameters such as the curvature ratio (R/a), the modular ratio (E_1/E_2), the thickness ratio (a/h), the lay-up angle, the number of layer and the boundary condition on the free vibration behaviour of the laminated composite curved/flat panels are computed in MATLAB based on the developed mathematical model and ANSYS.

3.5.1 Free vibration analysis using HSDT model

The non-dimensional fundamental frequency responses are obtained for three cross-ply [($0^0/90^0$), ($0^0/90/^00^0$) and ($0^0/90^0/90^0/0^0$)] laminated composite flat panel by varying thickness ratios (a/h=5, 10, 20, 50, 80 and 100) and modular ratios (E_1/E_2=5, 10, 15, 20, 25 and 30). The non-dimensional fundamental frequency responses are plotted in Figure 3.6 and Figure 3.7, for CCCC and CFFF support conditions, respectively. It can be easily visualize from the figures that increase in the thickness ratio and the modular ratio, the structural responses are increases. This is because of the fact that, as a/h ratio increases the panel becomes thin and the frequency value increases for thin structural component. Similarly, when the modular ratio increases the material orthotropicity of the laminated structure also increases and the frequency value shows an obvious behaviour. It is also noted that, ϖ is higher in case of CCCC supports in comparison to CFFF support and the vibration frequency increases with number of layers and the support conditions affects the responses greatly.

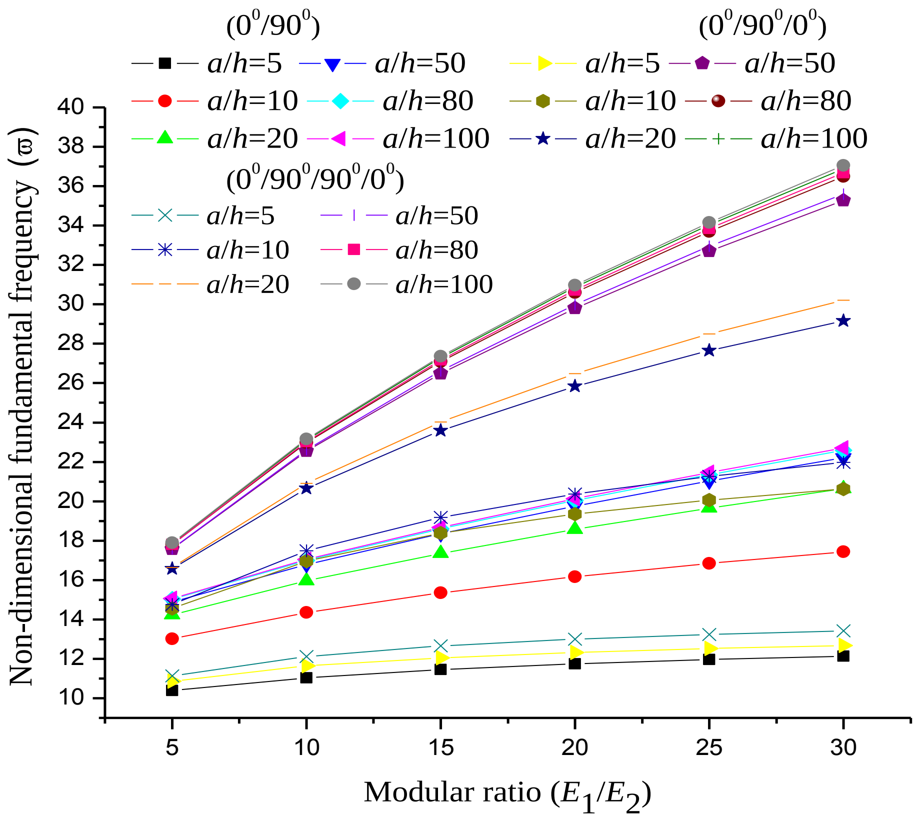


Figure 3.6 Non-dimensional fundamental frequency responses of clamped laminated composite flat panel

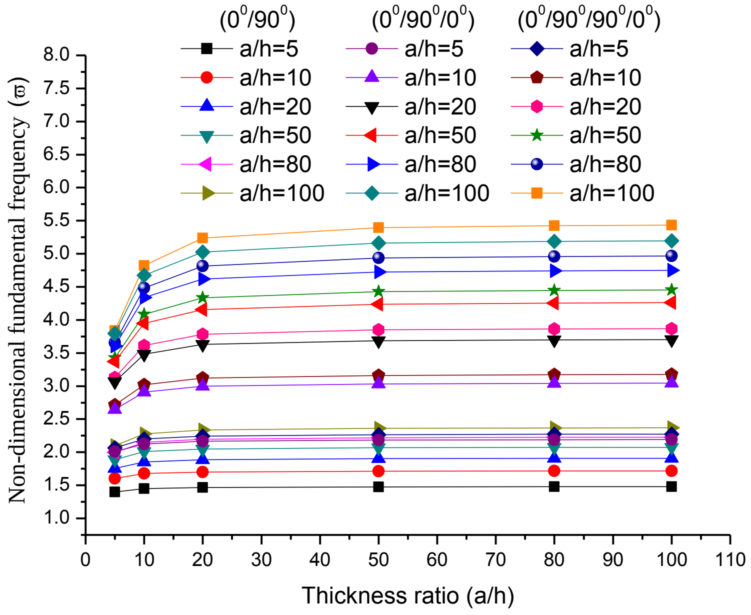


Figure 3.7 Non-dimensional fundamental frequency responses of CFFF laminated composite flat panel

The non-dimensional fundamental frequency responses of square clamped cross-ply [($0^0/90^0$), ($0^0/90/^00^0$) and ($0^0/90^0/90^0/0^0$)] laminated composite cylindrical panel has been obtained by varying thickness ratios (*a*/*h*=5, 10, 20, 50, 80 and 100) and curvature ratios (*R*/*a*=5, 10, 15, 20, 25 and 30) and the responses are plotted in Figure 3.8. It is clear from the figure that, the non-dimensional fundamental frequencies increases with increase in *a*/*h* and decrease in *R*/*a*. It is also visualise that lay-up scheme has significant effect on the responses of the structure.

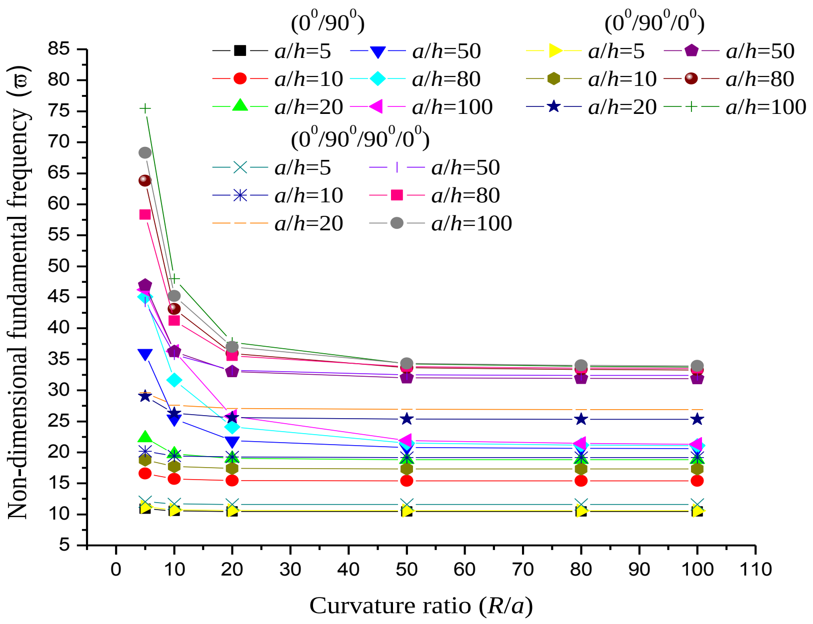


Figure 3.8 Non-dimensional fundamental frequency responses of clamped laminated composite cylindrical panel

The effects of geometrical parameters on the free vibration responses of laminated panels have been studied for different shell geometries as discussed in earlier paragraphs. Figure 3.9, Figure 3.10 and Figure 3.11 are showing the free vibration behaviour of the spherical, elliptical and hyperboloid laminated composite panels, respectively. The responses are obtained for four different types of support conditions i.e., SSSS, CCCC, CFFF and SCSC. It can be seen that the non-dimensional fundamental frequencies are showing higher values for CCCC support whereas it is minimum for CFFF condition. This is because of the fact that, the responses are following an increasing trend with increase in number of constraints. It is observed that the spherical, hyperboloid and elliptical laminated curved panels are mostly following the same type of trend for various geometrical and support conditions.

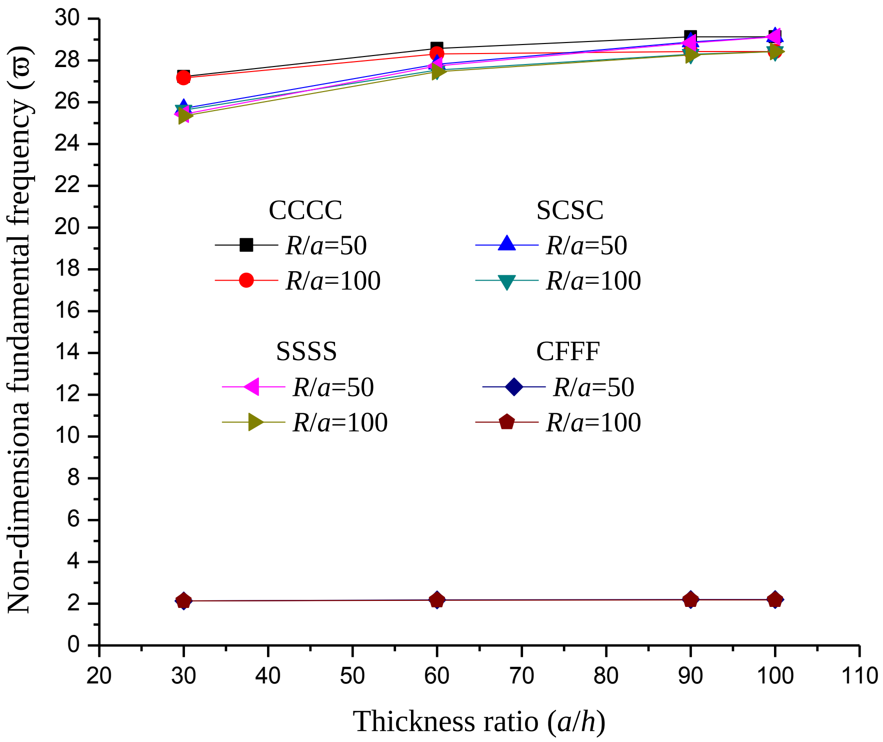


Figure 3.9 Non-dimensional fundamental frequency responses of angle-ply $(\pm45^0)_5$laminated composite spherical panel for different support condition

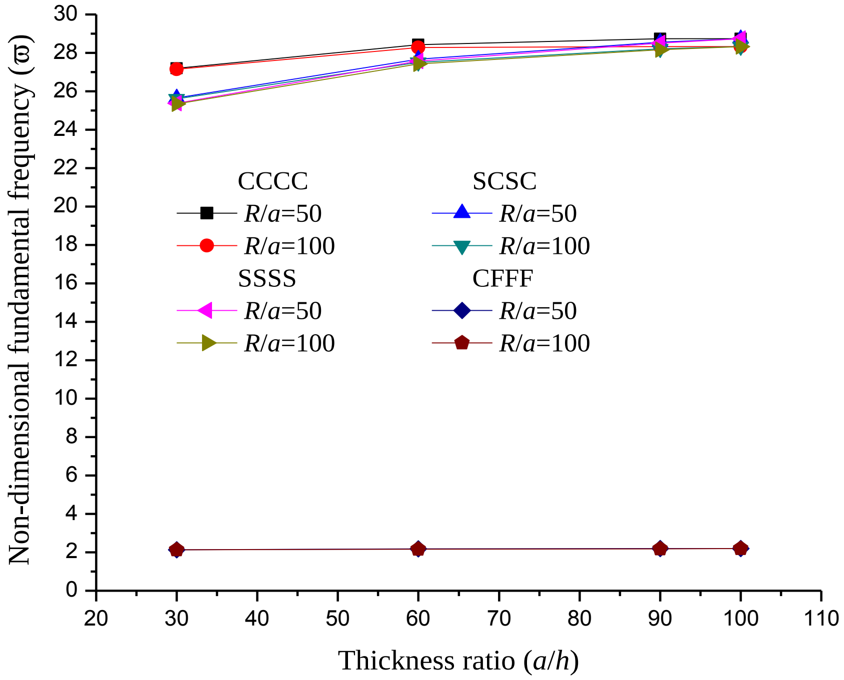


Figure 3.10 Non-dimensional fundamental frequency responses of angle-ply $(\pm 45^0)_5$ laminated composite elliptical panel for different support condition

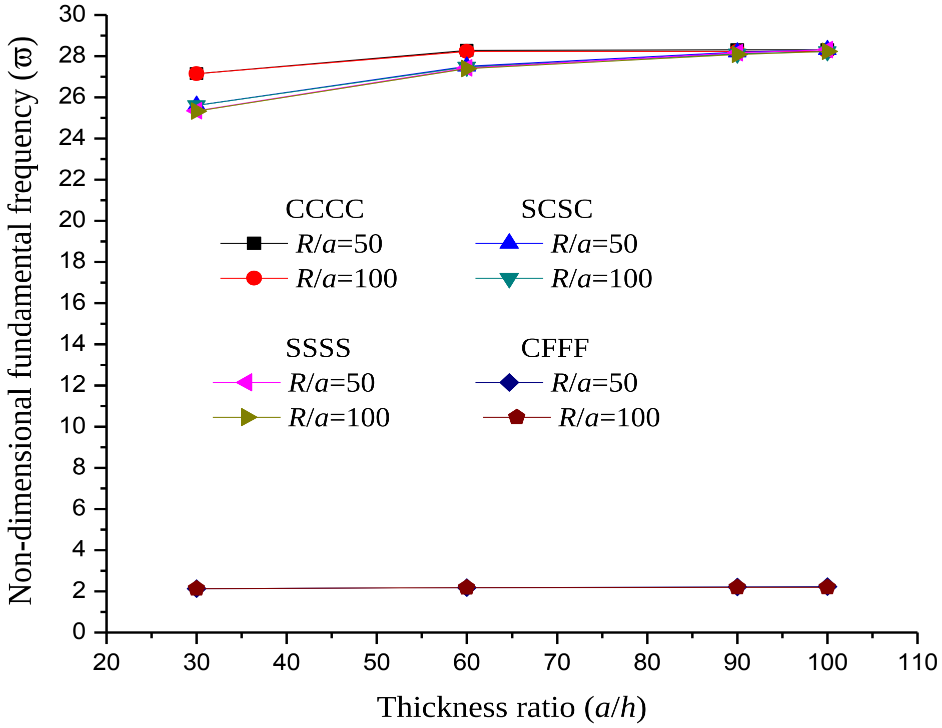


Figure 3.11 Non-dimensional fundamental frequency responses of angle-ply $(\pm 45^0)_5$ laminated composite hyperboloid panel for different support condition

3.5.2 Free vibration analysis using ANSYS model

In addition to above, some more new examples of flat/spherical/cylindrical panel examples have been solved by varying the geometrical parameters using the material properties same as discussed in the subsection 3.4. The free vibration responses of anti-symmetric cross-ply $(0^0/90^0)_2$ laminated cylindrical panel is analysed for two different supports SSSS and CCCC and presented in Figure 3.12. In this analysis other geometrical parameters are also varied for the computational purpose such as four thickness ratios (a/h=10, 20, 50 and 100) and five curvature ratios (R/a=5, 10, 20, 50 and 100) of the cylindrical panel. It is interesting to note that the responses are following the same types of trend as in the earlier examples under the subsection 3.5.1.

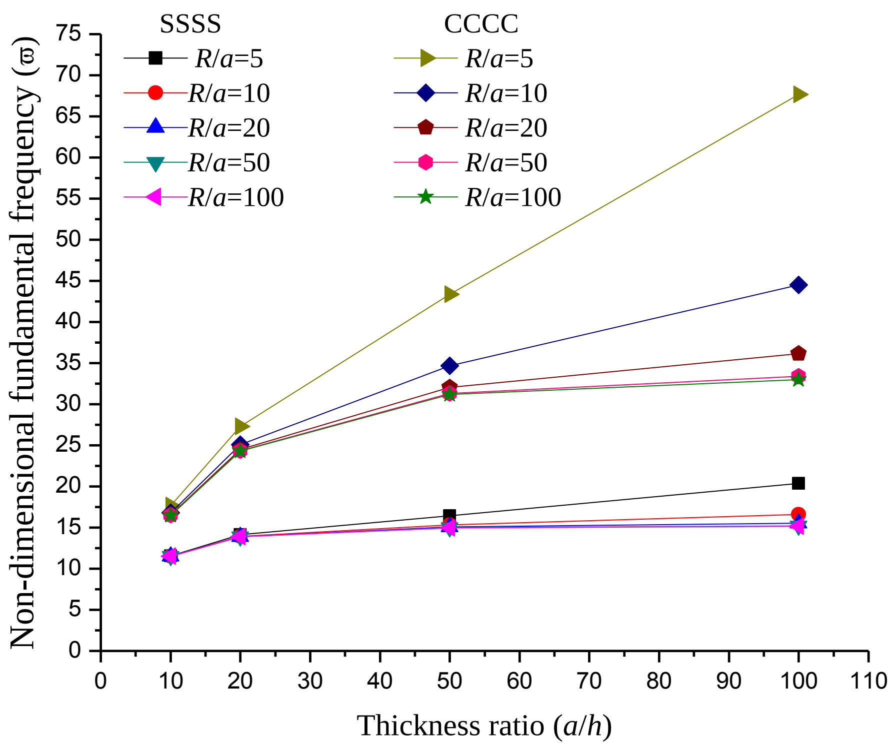


Figure 3.12 Non-dimensional fundamental frequency of cross-ply $(0^0/90^0)_2$ laminated composite cylindrical panel

In this example, the variation of non-dimensional fundamental frequency responses of the laminated composite angle-ply $(\pm 30^0)_2$ cylindrical panel is analysed by taking the same material properties, geometrical parameters and support conditions. The non-dimensional fundamental frequency responses are plotted in Figure 3.13. It is also interesting to note that the vibration behaviour of angle-ply laminated panel follows the same line as like in case of the cross-ply laminates.

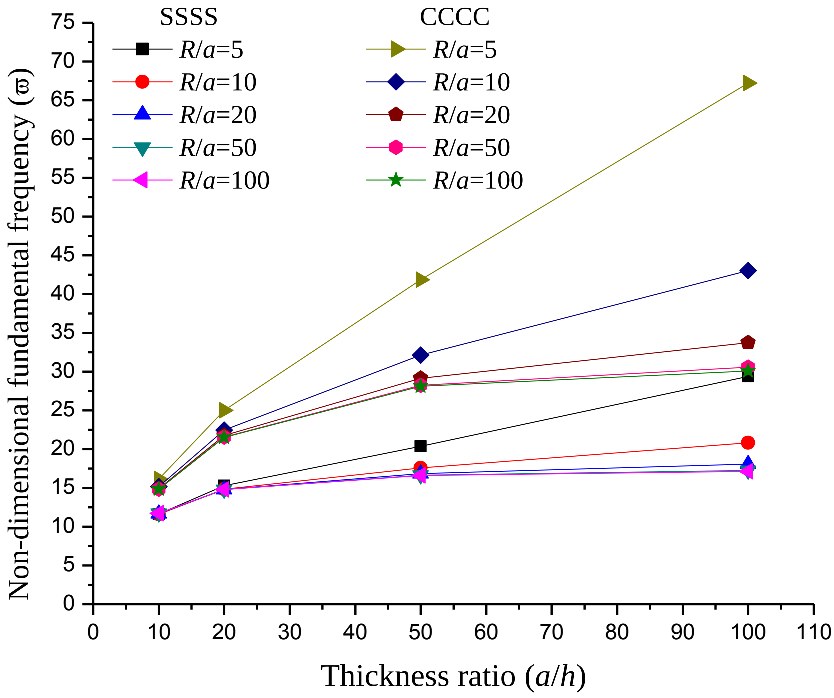


Figure 3.13 Non-dimensional fundamental frequency of angle-ply $(\pm 30^0)_2$ laminated composite cylindrical panel

In addition to the above, laminated composite spherical panel is analysed for five different thickness ratios (*a*/*h*=5, 10, 20, 50 and 100) five aspect ratios (*a*/*b*=1, 2, 5, 10 and 15) and five curvature ratios (*R*/*a*=5, 10, 20, 50 and 100). The analysis have been done in ANSYS for symmetric cross-ply $(0^0/90^0)_S$ and angle-ply $(\pm 45^0)_S$ laminated SSSS, CCCC and CFFF spherical panels. The non-dimensional fundamental frequency responses are obtained and shown in Figure 3.14 and Figure 3.15 and Figure 3.16.

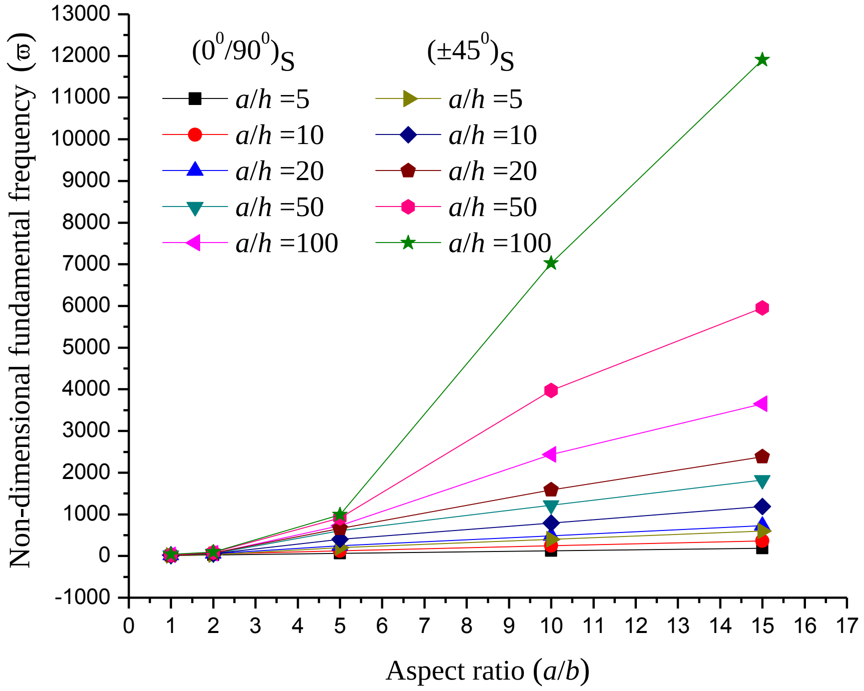


Figure 3.14 Non-dimensional fundamental frequency responses of simply supported laminated composite spherical shell panel (R/a=10)

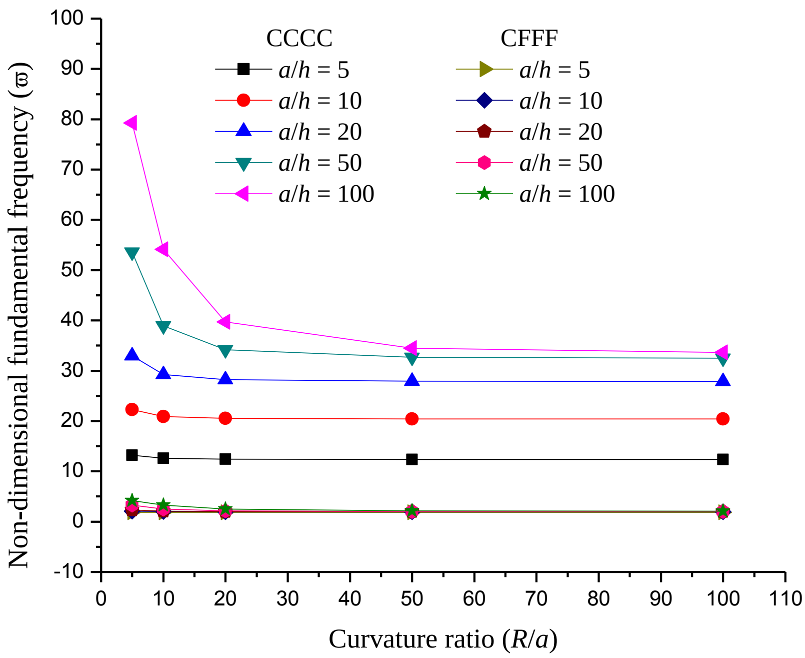


Figure 3.15 Non-dimensional fundamental frequency responses of cross-ply $(0^0/90^0)_S$ laminated composite spherical shell panel (a/b=1)

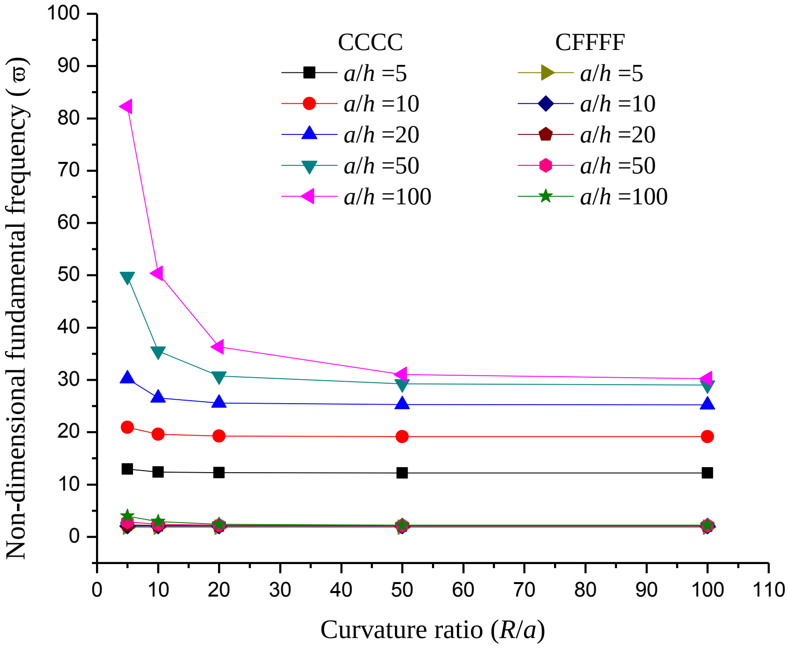


Figure 3.16 Non-dimensional fundamental frequency responses of angle-ply $(\pm 45)_S$ laminated composite spherical shell panel (a/b=1)

Figure 3.17 presents the free vibration responses of cross-ply $(0^0/90^0)_2$ laminated composite flat panel for SSSS and CCCC supports. The responses are obtained by varying the thickness ratio (a/h=10, 20, 50 and 100) and the modular ratio (E_1/E_2=10, 20, 30, 40 and 50).

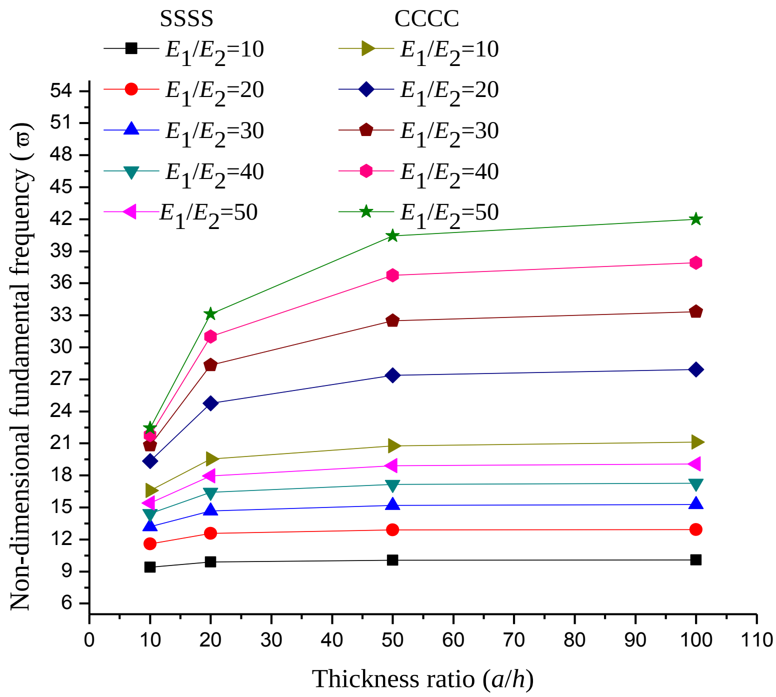


Figure 3.17 Non-dimensional fundamental frequency of cross-ply $(0^0/90^0)_2$ laminated composite flat panel

In this current examples, the effect of number of layers and support conditions (CCCC and CSCS) on the vibration behaviour is examined and presented in Figure 3.18 and Figure 3.19. In this numerical examples, six thickness ratios (a/h = 5, 10, 15, 20, 50, and 100), five modular ratios (E_1/E_2=10, 20, 30, 40 and 50) and different lay-up scheme [$(0^0/90^0)_2$, $(0^0/90^0)_3$, $(0^0/90^0)_5$] is used for the computation using the material property M2 from the Table 3.1. It is clear from the figure that, the thickness ratio and the modular ratio increases the non-dimensional fundamental frequency increases and this behavior is expected for any laminated structure. It is noted that the numbers of layers of the flat panel increases the non-dimensional fundamental frequency of the flat panel also increases.

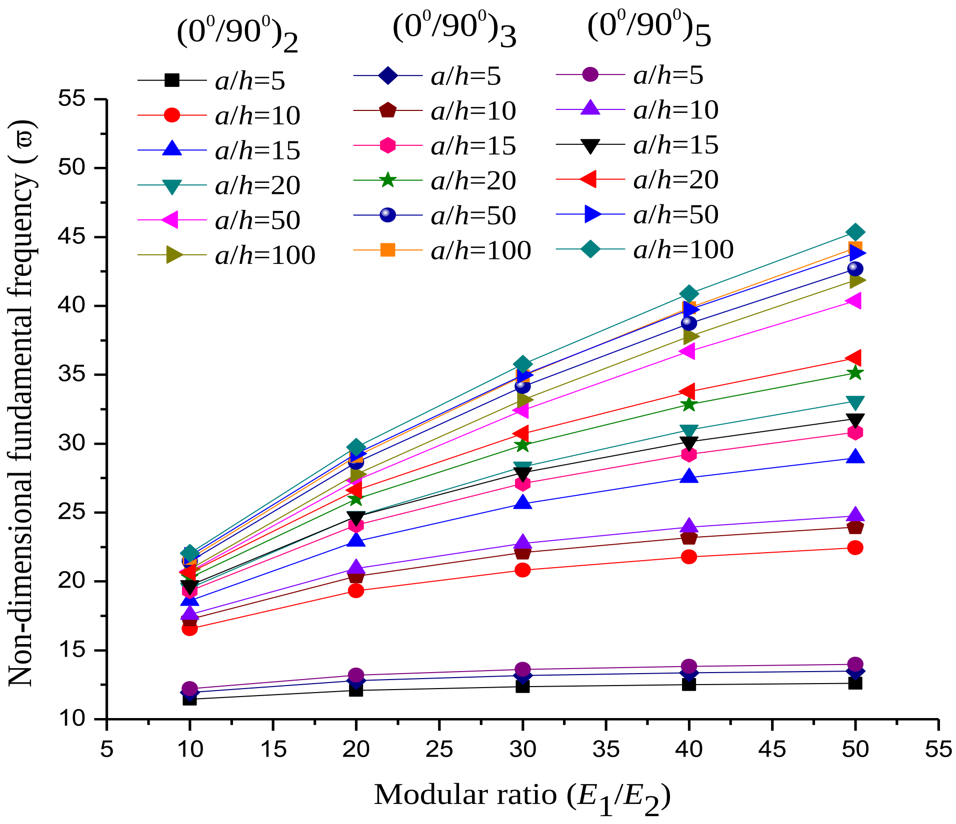


Figure 3.18 Non-dimensional fundamental frequency of clamped laminated composite flat panel

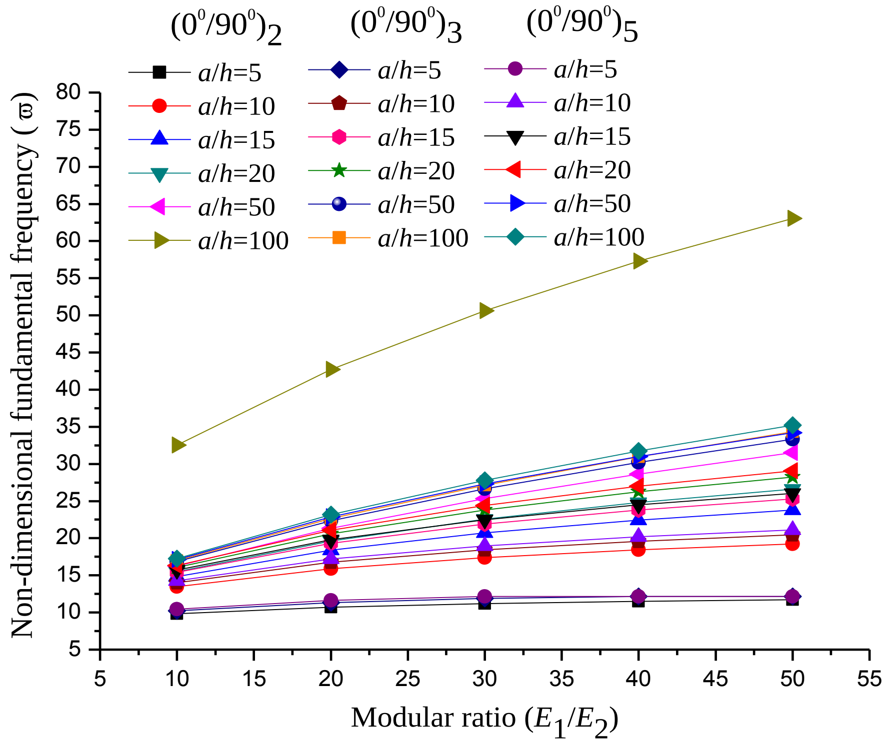


Figure 3.19 Non-dimensional fundamental frequency of CSCS laminated composite flat panel

3.6 Conclusions

In this chapter, the free vibration behaviour of doubly curved laminated composite shell panel is investigated using the developed general shell panel model in Chapter 2. The free vibration of shell panel are computed using the eigenvalue formulation and are solved using the FEM code developed in MATLAB and APDL code in ANSYS. The effects of the curvature ratio, the thickness ratio, the aspect ratio, the modular ratio, the staking sequence and different support conditions on the fundamental frequency of various geometries are examined. Based on the numerical results the following conclusions are drawn.

- The validation and convergence study shows that the present developed model is capable to solve different free vibration problem with ease.
- The non-dimensional fundamental frequency responses are increases with increase in the modular ratio, the thickness ratio and the aspect ratio whereas it decreases with increase in the curvature ratio.
- It is noted that, the thickness ratio, the modular ratio, the curvature ratio, the aspect ratio and the support conditions affect the non-dimensional fundamental frequency considerably.
- The lay-up scheme has a significant effect on the non-dimensional fundamental frequency.

CHAPTER 4

BUCKLING ANALYSIS OF LAMINATED COMPOSITE SHELL PANELS

4.1 Introduction

In this chapter, the buckling strength of laminated shell panels has been studied using the proposed developed model under thermal/mechanical/thermo-mechanical environment. As discussed earlier in Chapter 1, the structural components of aerospace, launch vehicles, rockets etc. are made up of laminated composites normally having shell type geometries which are often subjected to intense thermal/mechanical/thermo-mechanical loading due to the aerodynamic heating during their service life. The temperature increases in the structural component induces buckling/instability which in turn degrades the performance of the whole structure. It is well known that the buckling doesn't mean the ultimate failure of the structural components and they are still capable to carry extra amount of load beyond the buckling point without failure. It is also important to mention that the geometric strain associated with the buckling phenomena is nonlinear in nature. From the design and analysis point of view, it is important to know the exact value of buckling strength in thermal/mechanical/thermo-mechanical environment. In addition to this, the mathematical model has to be sufficient enough to incorporate the true geometric alterations.

In this present chapter, the thermal/mechanical/thermo-mechanical buckling load parameter of the shell panel of various geometries has been obtained. It is also necessary to mention that, to explore the original strength of laminated structures, the mathematical model is developed in the framework of the HSDT by taking the nonlinearity in Green-Lagrange sense to incorporate the true geometrical distortion in geometry. In addition to that all the terms evolved in the formulation have been incorporated in the mathematical model to achieve a realistic case.

4.2 Governing Equations and Solution

The system governing equilibrium equations of a buckled laminated composite shell panel is obtained initially by dropping the appropriate terms from the Eq. (2.20) and that may be expressed as:

$$\left\{\left[K_S\right]-\lambda_{cr}[K_G]\right\}\{\delta\}=0 \tag{4.1}$$

where, $\left[K_S\right]$ is the stiffness matrix, $\left[K_G\right]$ is the global stiffness matrix, λ_{cr} is the critical buckling load parameter and $\{\delta\}$ is the displacement vector.

The Eq. (4.1) has been solved using the steps as discussed in the Chapter 2 and the desired responses are obtained.

4.3 Results and Discussions

In this section, some numerical examples have been solved to obtain the thermal/mechanical/thermo-mechanical buckling load parameter of laminated composite shell panels by taking nonlinear geometry matrix. In order to do so, a finite element based code has been prepared in MATLAB 7.10 using the developed mathematical model. As a first step, the validation and accuracy of the present developed code has been examined by comparing the results with those available in literature and ANSYS model too. In order to demonstrate the efficacy of the present numerical model a detailed parametric study has been carried out for the curved/flat panel and the results obtained are presented and discussed. It is observed that the responses obtained using the mathematical model and the FE tool are in good agreement with the available published literature. The effects of different combinations parameters like the curvature ratio (R/a), the thickness ratio (a/h), the modular ratio (E_1/E_2), the lay-up scheme and the support condition on the composite shell panel responses are also studied. For the computational purpose, the following composite material properties have been used.

Material-1 (M1): $E_1/E_0 = 21$; $E_2/E_0 = 1.7$; $G_{12}/E_0 = G_{13}/E_0 = 0.65$; $G_{23}/E_0 = 0.639$; $\nu_{12} = 0.21$; $\alpha_{11} = -0.21\ \alpha_0$; $\alpha_{22} = 16\ \alpha_0$; $\alpha_0 = 1\times10^{-6}/{}^0\mathrm{C}$

Material-2 (M2): $E_1/E_2 = 15$; $G_{12} = G_{13} = 0.5000E_2$; $G_{23} = 0.3356E_2$; $\upsilon_{12} = \upsilon_{13} = 0.30$; $\upsilon_{23} = 0.49$; $\alpha_1/\alpha_2 = 0.015\alpha_0$; $\alpha_0 = 1\times10^{-6}/{}^0\mathrm{C}$

Material-3 (M3): $E_1/E_2 = 25$; $G_{12}/E_2 = G_{13}/E_2 = 0.5$; $G_{23}/E_2 = 0.2$; $\nu_{12} = 0.25$

4.4 Convergence and Validation Study of Buckling

The validation and the convergence of the present developed model have been checked by examining different numerical examples. As discussed earlier, the responses are obtained by solving numerically through the developed computer code and APDL code in ANSYS and the responses are compared with those published literature. Based on the convergence, a (6×6) mesh is adopted for the computation of the thermal and mechanical buckling responses for throughout the analysis. It is also important to mention that the mechanical buckling load has been obtained in APDL code (ANSYS) is converging at a (16×16) mesh in throughout the analysis. The non-dimensional forms of the critical buckling temperature and the critical buckling loads are obtained using the formulae, $\lambda_{cr} = \alpha_0 T_{cr} 10^3$ and $N_x = N_{xx} a^2 / E_2 h^3$, respectively. It is same in throughout the analysis if it is not stated elsewhere.

Figure 4.1 shows the comparison and convergence behaviour of non-dimensional buckling temperature parameter with different mesh divisions for a square thin (*a*/*h* = 80 and 100) simply supported angle-ply $(\pm 45^0)_3$ laminated composite flat panel subjected to uniform temperature field. The critical buckling temperature results are obtained using the geometrical properties same as in the reference and M2. The figure shows that, the responses obtained from the present computer code (MATLAB) and the APDL code are converging well with the mesh refinement and a very small difference exists with Chang and Leu [34] and Shen [39].

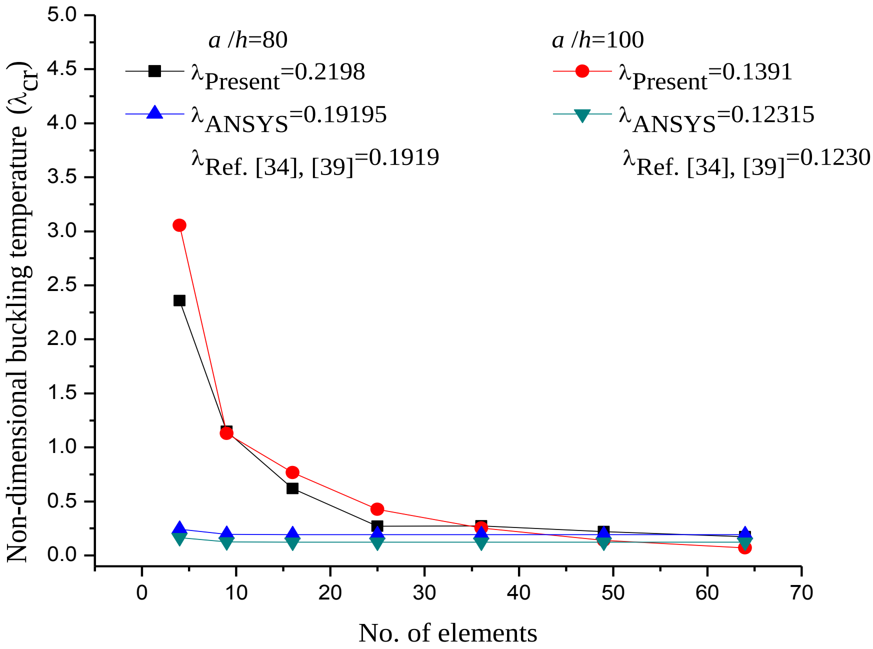


Figure 4.1 Convergence study of non-dimensional buckling temperature of simply supported angle-ply $(\pm 45^0)_3$ laminated composite flat panel

Now, one more case of buckling behaviour is being analysed for the buckling temperatures of laminated composite flat panel. In order to obtain the temperature responses, the present developed HSDT mathematical model is employed where the geometry matrix is evaluated based on the Green-Lagrange type of nonlinear kinematics. The numerical investigations have been done using the same geometrical and material properties (M1) of Kant and Babu [48] and presented in Table 4.1.

Table 4.1 Comparison of non-dimensional buckling temperature of simply supported angle-ply laminated flat panel

a/h	**Present**	**Source**	**Lay-up**
10	0.073	0.079 [30]	$(\pm 15^0)_5$
100	0.0016	0.0015 [30]	$(\pm 30^0)_5$

In this paragraph the convergence and validation study of mechanical buckling load has been discussed based on the APDL code and computer code developed using the proposed HSDT mathematical model. In this computation, a square simply supported laminated cylindrical panel model has been developed for two anti-symmetric cross-ply $[(0^0/90^0)_2$ and $(0^0/90^0)_5]$ using APDL code in ANSYS. The material (M3) and geometrical properties have been taken same as Nguyen-Van *et al*. [17]. The responses are plotted in Figure 4.2. The critical buckling load parameters with mesh refinement are obtained for uniaxial loading condition. It is understood that the present results are converging well and the differences are very small with the references. Similarly, the mechanical buckling load parameter has been computed using HSDT model as discussed initially in this paragraph and the responses are presented in Table 4.2. The responses are obtained for a simply supported laminated flat panel using the material and geometrical properties same as the references.

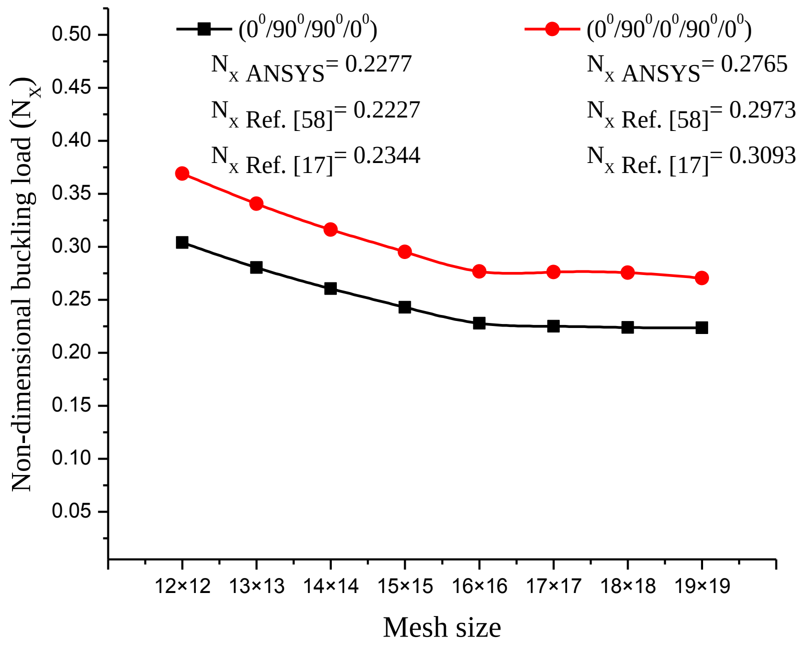


Figure 4.2 Convergence study of non-dimensional buckling load of simply supported laminated composite cylindrical shell panel (R/a = 2 and a/h=5)

In the following section, some new results are computed for different geometrical parameters and their effects on buckling (thermal/mechanical/thermo-mechanical) load is discussed. The responses are obtained using both APDL code and MATLAB code for various combinations and the effects are discussed in details.

Table 4.2 Comparison of non-dimensional buckling load of square simply supported cross-ply laminated composite flat panel

a*/*h	**Present**	**Source**	**Lay-up**
5	0.2765	0.2973 [58]	($0^0/90^0/0^0/90^0/0^0$)
10	5.3416	5.3044 [31]	($0^0/90^0/90^0/0^0$)

4.5 Numerical Examples

As discussed earlier in Chapter 3, the laminated structures of various geometries of flat/curved laminated composite shell panels like, spherical, cylindrical, elliptical, hyperboloid and flat panels are computed. The effect of various geometric and material properties on the buckling (thermal/mechanical/thermo-mechanical) behaviour are discussed for each type of shell panel by solving different numerical examples.

4.5.1 Thermal buckling analysis using HSDT model

As a first step, the critical buckling temperature parameter of laminated composite flat panel and the effect of number of layers (two and six), five thickness ratios (*a*/*h*), lay-up sequence (cross-ply and angle-ply) and three support conditions (SSSS, CCCC and CFFF) are computed and presented in Figure 4.3. The material properties used for the computation are same as Shen [39]. It is clear from the figure that, the non-dimensional buckling temperature parameter of cross-ply laminated panel is less than that of the angle-ply as expected. But, it is interesting to note that the buckling temperature parameter increases as the panel becomes thinner i.e., *a*/*h* increases due to the fact that the small strain but large deformation regime the behaviour may not follow any monotonous trend. It is noticed that the non-dimensional buckling temperature parameter is increases with an increase in the thickness ratios (*a*/*h*) and as the lamination scheme changes from cross-ply to angle-ply. It is also noted that the thermal critical buckling load parameter is higher for CCCC support in comparison to other two supports, CFFF and SSSS. In both of the figures it is clearly observed that the buckling load parameter increases very slowly from *a*/*h*=5 to 10 and then the load increases suddenly afterword. It is because of the fact as the a/h ratio increases the panel becomes thin and the buckling load strength may deviate from the expected line.

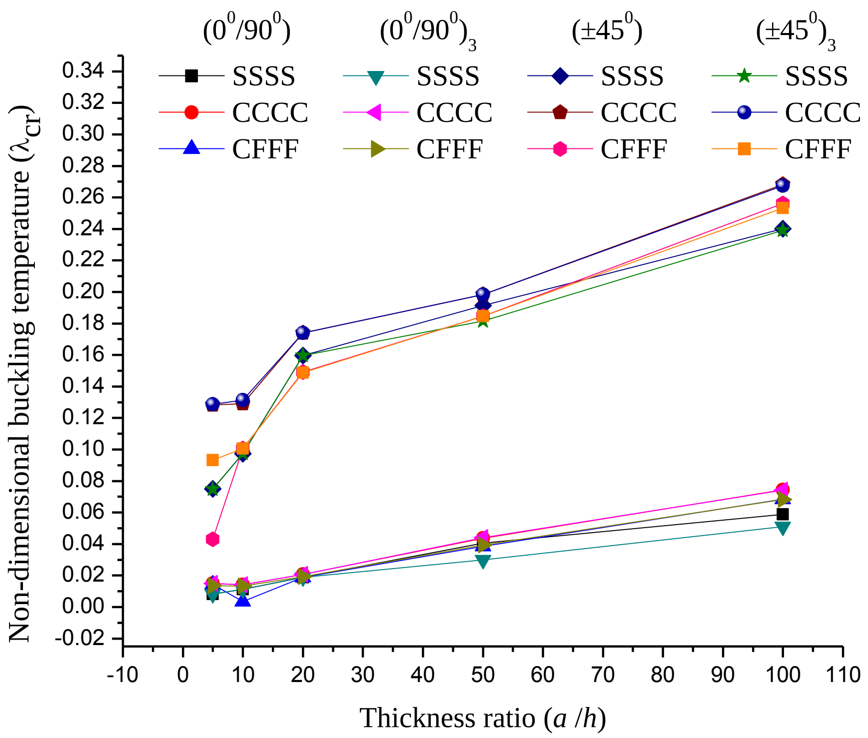


Figure 4.3 Variation of buckling temperature parameter of cross-ply and angle-ply laminated composite flat panel

Similarly, the critical buckling temperature parameter has been obtained for a square cross-ply $(0^0/90^0)_3$ and angle-ply $(\pm15^0)_3$ laminated composite cylindrical panel for three curvature ratios with two different support conditions (SSSS and CFFF). The buckling responses are computed for six thickness ratios (a/h = 20, 30, 40, 50, 100 and 120) and three curvature ratios (R/a = 5, 10 and 100) and presented in Table 4.3 using the material properties as in Shen [39]. The table depicts that the buckling temperature parameter increases as the curvature ratio increases and the angle-ply has higher thermal buckling strength in comparison to cross-ply. It is observed that the buckling load parameter increases as the curvature ratio and the thickness ratio increases for cross-ply laminations whereas the buckling temperature decreases for angle-ply laminations as the a/h ratio increases. In addition to above the thermal buckling load parameter is showing higher value for CFFF support in comparison to SSSS support. It is also noted that angle-ply laminations are showing higher thermal buckling load as compared to the cross-ply laminations hence, the angle-ply laminates more are preferred for thermal shielding cases.

Table 4.3 Variation of buckling temperature parameter of cross-ply $(0^0/90^0)_3$ and angle-ply $(\pm15^0)_3$ laminated composite cylindrical panel

			a/h				
		R/a	**30**	**40**	**50**	**100**	**120**
$(0^0/90^0)_3$	**SSSS**	**5**	0.0268	0.0289	0.0295	0.034	0.0396
		10	0.0271	0.0292	0.0298	0.0374	0.0651
		100	0.0279	0.0299	0.0306	0.0497	0.1119
	CFFF	**5**	0.0206	0.0288	0.0298	0.0305	0.0627
		10	0.0265	0.029	0.03	0.0315	0.0419
		100	0.0272	0.0304	0.0309	0.032	0.034
$(\pm15^0)_3$	**SSSS**	**5**	0.9221	0.8517	0.8264	0.5421	0.2412
		10	0.9277	0.8615	0.8628	0.6412	0.2452
		100	0.9281	0.8652	0.8635	0.6822	0.5258
	CFFF	**5**	0.875	0.8384	0.797	0.227	0.1528
		10	0.8752	0.8463	0.8094	0.2288	0.1535
		100	0.8758	0.847	0.8182	0.3764	0.1602

Now, in this section the thermal buckling strength of laminated angle-ply $(\pm45^0)_5$ cylindrical panel has been studied using the developed mathematical model by varying five thickness ratios (a/h = 5, 10, 20, 50 and 100) and two curvature ratios (R/a = 2 and 100). The responses are obtained using the material properties same as Shen [39] and presented in Table 4.4. It is noticed from the table that, the critical buckling temperature parameter increases with increase in the thickness ratio and decrease in the curvature ratio. The responses are not following a monotonous trend and reverts from the expected line with the thickness ratio.

This is because of the fact that the thin structure may not follow a monotonous trend of results due to severity in geometrical distortion.

Table 4.4 Non-dimensional buckling temperature of clamped square angle-ply $(\pm 45^0)_5$ laminated cylindrical shell panel

a*/*h	***R*/*a***	
	2	**100**
5	0.0376	0.0370
10	0.0351	0.0349
20	0.0503	0.0503
50	0.0876	0.0641
100	0.0871	0.0687

In addition to all the discussion made in above paragraphs the efficacy of the present developed model is being analysed by conducting a comparative study of different geometries such as spherical, elliptical and hyperboloid. The critical buckling temperature responses are obtained by varying thickness ratios (a/h = 30 and 90) and curvature ratios (R/a = 50 and 100) for SSSS, CCCC, SCSC and CFFF condition. The critical buckling temperature parameter responses are obtained using the material properties same as Shen [39] and tabulated in the Table 4.5. It is observed that the response are following the same trend as discussed for earlier cases. It is observed that the buckling load parameter for hyperboloid panel is higher as compared in other two geometries i.e., spherical and elliptical.

Table 4.5 Critical buckling temperature responses of cross-ply $(0^0/90^0)_5$ laminated composite spherical, hyperboloid and elliptical panel for different support condition

Geometry of the panel	**Support condition**	***R*/*a***			
		50		**100**	
		a*/*h			
		30	**90**	**30**	**90**
Spherical	**CCCC**	0.0276	0.0296	0.0273	0.0544
	SCSC	0.0273	0.0306	0.0273	0.0475
	SSSS	0.024	0.0374	0.025	0.0452
	CFFF	0.0261	0.0306	0.0244	0.041
Hyperboloid	**CCCC**	0.0282	0.0388	0.0057	0.0057
	SCSC	0.0202	0.0347	0.0057	0.0057
	SSSS	0.0258	0.0306	0.0253	0.0296
	CFFF	0.0251	0.0297	0.0186	0.0392
Elliptical	**CCCC**	0.022	0.022	0.0179	0.0179
	SCSC	0.022	0.022	0.0179	0.0187
	SSSS	0.025	0.0268	0.0253	0.0261
	CFFF	0.0247	0.027	0.0255	0.0512

4.5.2 Thermal buckling analysis using ANSYS model

Based on the objective of the study all different types of laminated composite panel problems are solved using both MATLAB code and ANSYS. The thermal buckling behaviour of various geometries are being analysed based on the developed mathematical model through the MATLAB code up to the subsection 4.5.1. This point onwards the thermal buckling behaviour of laminated flat panel has been analysed using APDL code in ANSYS for different parameters.

Figure 4.4 and Figure 4.5 are showing the thermal buckling behaviour laminated composite flat panels for two different support conditions (SSSS and CCCC) and two anti-symmetric lamination schemes [$(0^0/90^0)_2$ and $(\pm 45^0)_2$]. The desired responses are obtained using the M2 material properties for four thickness ratios (a/h = 10, 20 50 and 100) and five modular ratios (E_1/E_2 = 10, 15, 20, 25 and 30). In both the cases the buckling temperature follows the same trend i.e., it decreases with increase in the thickness ratio and decrease in the modular ratio. As earlier said as the thickness ratio (a/h) increases the structure becomes thin and loses its geometrical stability, similarly, change in the modular ratio also increase the orthotropicity of the material and which affects the structural stiffness greatly.

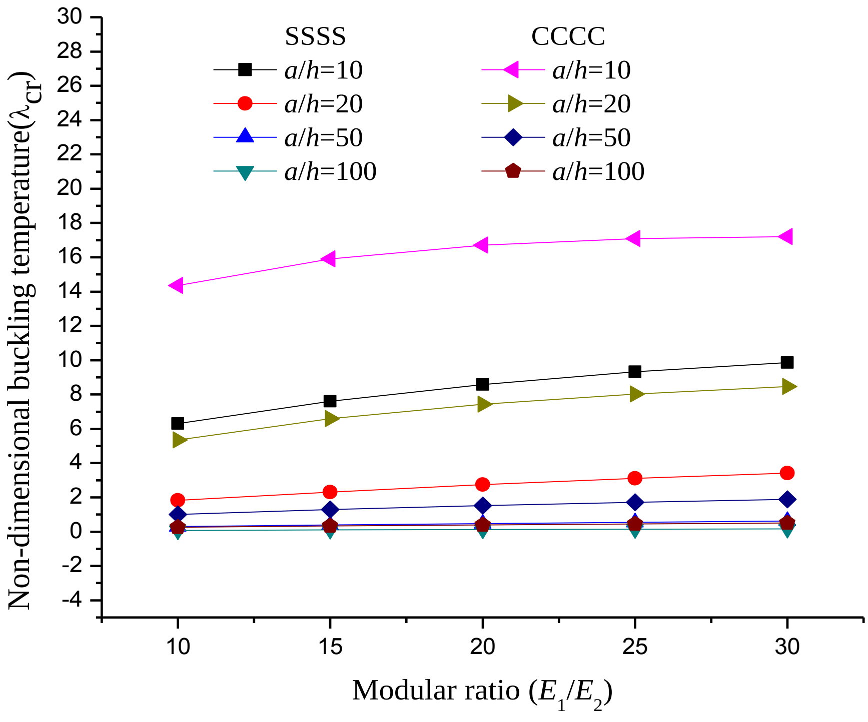


Figure 4.4 Non-dimensional buckling temperature for $(0^0/90^0)_2$ flat panel for different support condition

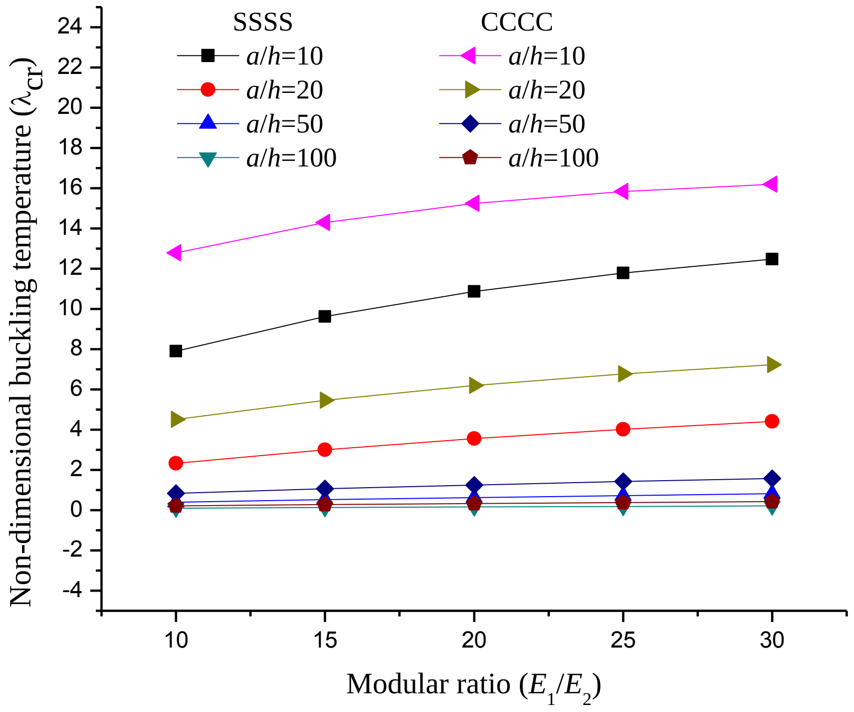


Figure 4.5 Non-dimensional buckling temperature for $(\pm45^0)_2$ flat panel for different support conditions

In continuation to the above cases, two more numerical experimentations have been done by solving two new examples for different parameters like six thickness ratios (a/h = 5, 10, 15, 20, 50 and 100), five modular ratios (E_1/E_2 = 1, 2, 3, 4 and 5) and three stacking sequences $[(\pm15^0)_3, (\pm30^0)_3$ and $(\pm45^0)_3]$. It is understood from the earlier examples that the number of layers have considerable effect on buckling strength. Hence, to demonstrate the same here the responses are obtained and presented in Figure 4.6 and Figure 4.7 for two support conditions, CCCC and CSCS, respectively. The material and the geometrical parameters are same as Chang and Leu [34]. In addition to the above, Figure 4.8 and Figure 4.9 depicted the first four buckling modes of SSSS and CCCC laminated anti-symmetric cross-ply $(0^0/90^0)_5$ flat panel.

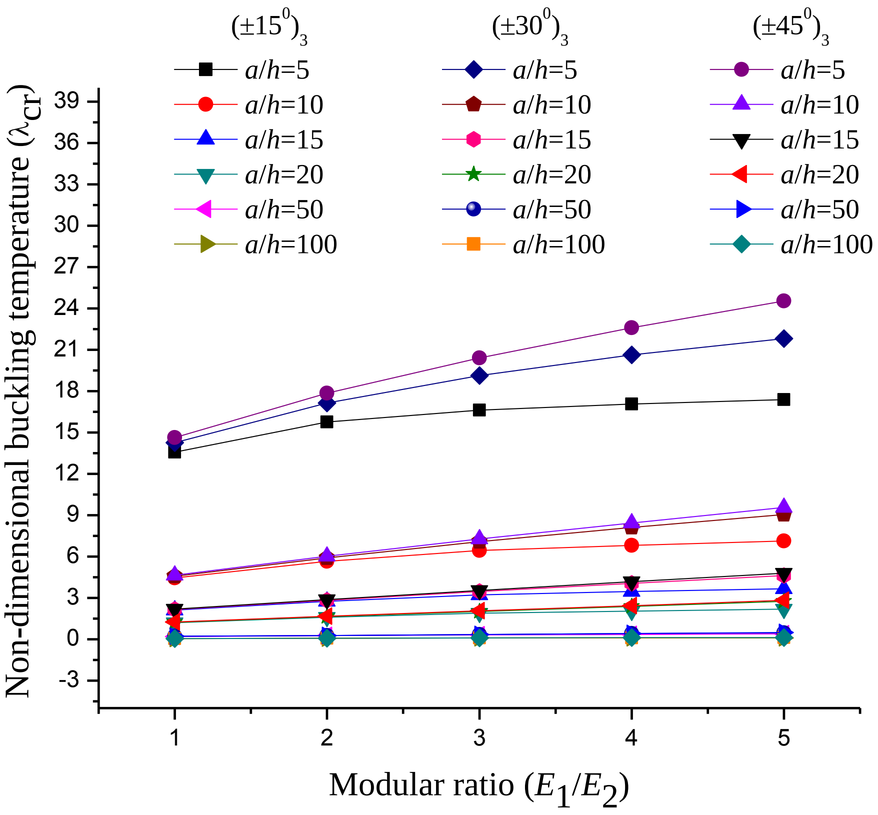


Figure 4.6 Non-dimensional buckling temperature of CCCC laminated composite flat panel

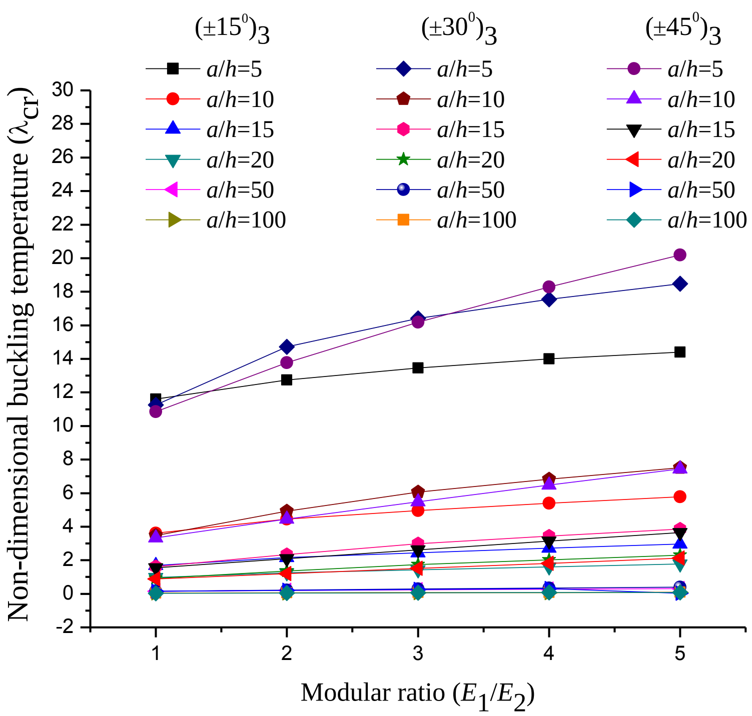
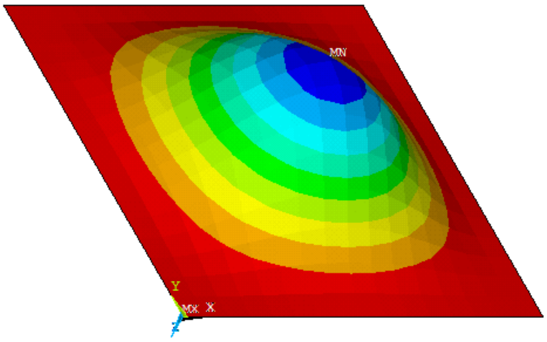
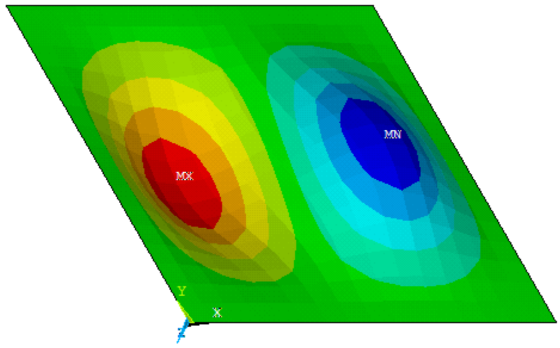


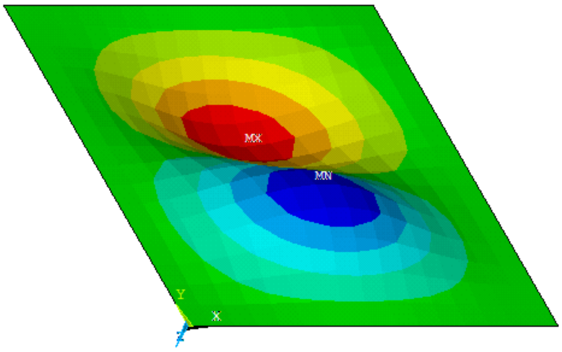
Figure 4.7 Non-dimensional buckling temperature of CSCS laminated composite flat panel



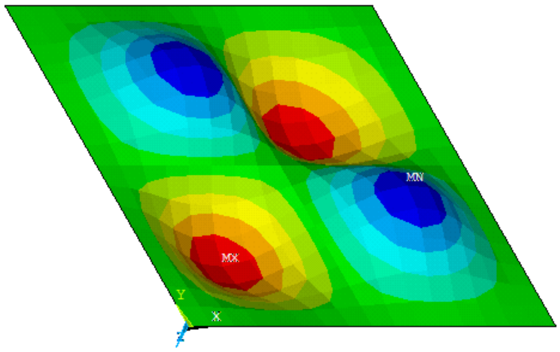
1st mode



2nd mode

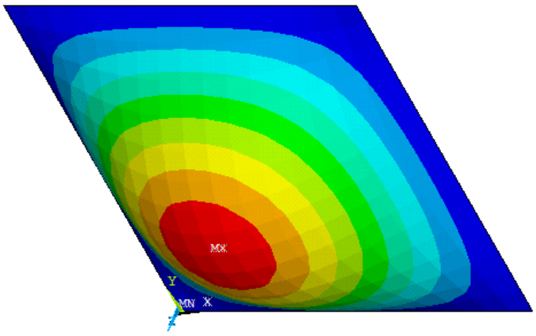


3rd mode

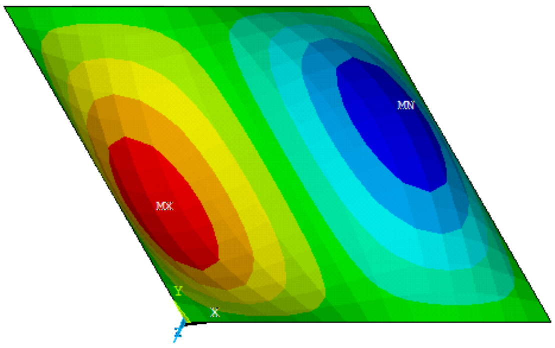


4th mode

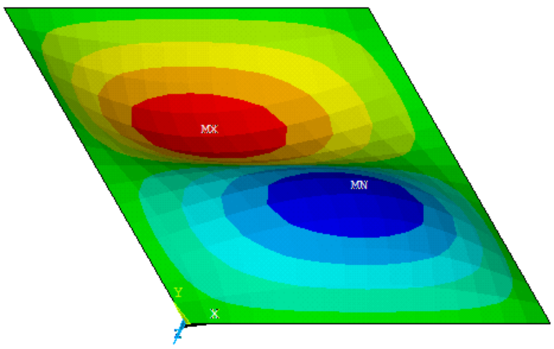
Figure 4.8 Mode shape of simply supported cross-ply $(0^0/90^0)_5$ laminated composite flat panel



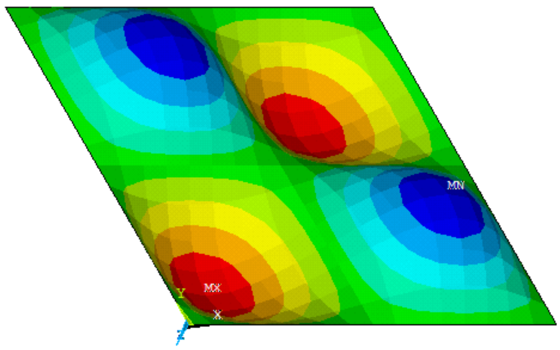
1st mode



2nd mode



3rd mode



4th mode

Figure 4.9 Mode shape of clamped cross-ply $(0^0/90^0)_5$ laminated composite flat panel

4.5.3 *Mechanical buckling analysis using HSDT model*

After analysing the thermal buckling behaviour of laminated composite panel the mechanical buckling strength of laminated panels have been discussed in this subsection. In order to do so, few examples have been solved and the buckling load parameters of laminated curved/flat panels are investigated for different loading (uniaxial or bi-axial) and geometrical and material properties. The responses are obtained using the present developed mathematical model and APDL code.

As a first step, the critical buckling load parameter is obtained for clamped cross-ply $(0^0/90^0)_5$ laminated shell (cylindrical, spherical, elliptical, hyperboloid and flat) panel using the material properties as Pandit *et al*. [71] under bi-axial loading. The responses are obtained for the curvature ratio (R/a=10), four thickness ratios (a/h=5, 10, 20 and 30) and presented in Table 4.6. The responses are indicating that the critical buckling load increases as thickness ratios increases. This is because of the fact that the thin flexible structures may not follow a specified trend of results due to the geometrical distortions are nonlinear in nature.

Table 4.6 Critical buckling load of clamped cross-ply $(0^0/90^0)_5$ laminated shell under bi-axial loading (E_1/E_2=40, R/a=10 and $N_x = N_{xx}a^2 / E_2 h^3$)

a/h	**Cylindrical**	**Spherical**	**Elliptical**	**Hyperboloid**	**Flat**
5	0.3315	0.3803	0.3801	0.3800	0.3800
10	1.2303	1.5501	1.5494	1.5493	1.5495
20	6.8042	9.1384	9.4004	9.2851	9.4025
30	13.8011	26.4035	24.7749	21.6513	29.8590

Similarly, one more example has been discussed for the mechanical buckling load parameter ($N_X = N_{XX} / (E_2 h)$) of simply supported angle-ply $(\pm 45^0)_5$ laminated curved/flat shell under uniaxial loading condition and the responses are shown in Table 4.7. The geometrical and material properties are taken as in Nguyen-Van *et al*. [17]. The responses are following the same type of trend as seen in previous case.

Table 4.7 Critical buckling load of simply supported angle-ply $(\pm 45^0)_5$ laminated shell under uniaxial loading (E_1/E_2=25, R/a=20 and $N_X = N_{XX} / (E_2 h)$)

a/h	**Cylindrical**	**Spherical**	**Elliptical**	**Hyperboloid**	**Flat**
5	0.0199	0.0187	0.0198	0.0116	0.0176
10	0.0203	0.0205	0.0191	0.0203	0.0178
20	0.0311	0.0311	0.0310	0.0309	0.0267
30	0.0441	0.0423	0.0438	0.0429	0.0373
40	0.0494	0.0318	0.0426	0.0471	0.0488

4.5.4 Mechanical buckling analysis using ANSYS model

Now, in this subsection, the flat/cylindrical panels are being analysed using APDL code in ANSYS and critical buckling load parameters are evaluated as discussed. To show the effect of loading type on non-dimensional buckling load parameter of symmetric cross-ply [($0^0/90^0/0^0$) and ($0^0/90^0/0^0/90^0/0^0$)] laminated composite flat panel is analysed using the material property as in Zhen and Wanji [33]. Figure 4.10 and Figure 4.11 presents the non-dimensional buckling load for different thickness ratios, modular ratios and CCSS support condition under bi-axial and uniaxial loading, respectively. It is clear from the figures that the non-dimensional buckling load increases, with decrease in the thickness ratio and increase in the modular ratio. The non-dimensional buckling load of the flat panel also increases as the numbers of layers increases which can be seen in the figures for different *a*/*h* values.

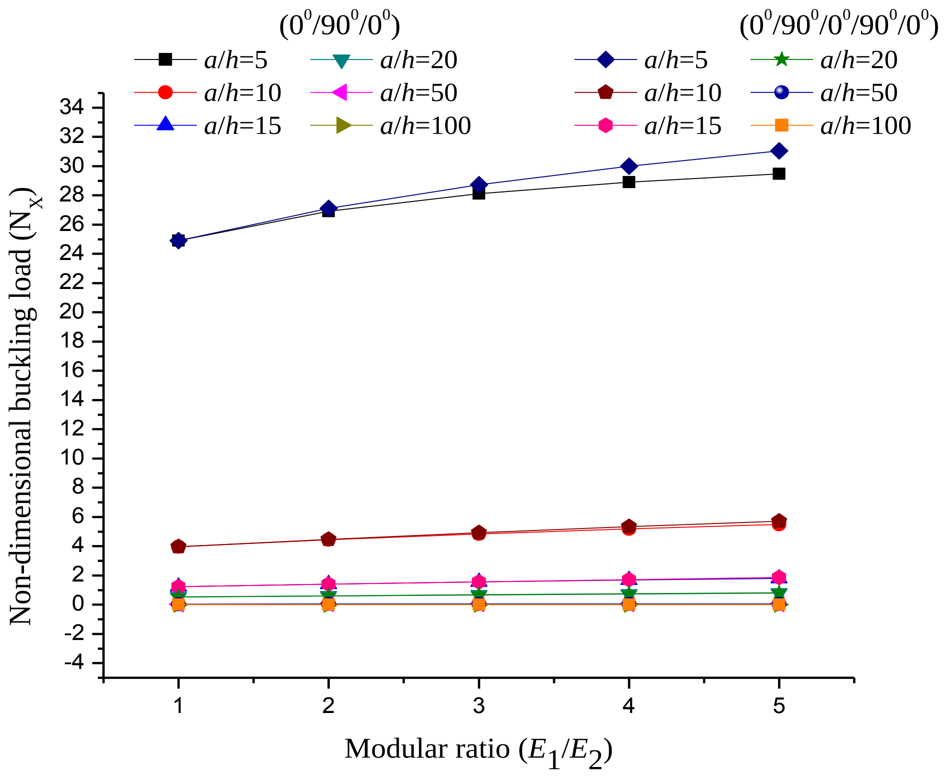


Figure 4.10 Non-dimensional buckling load of laminated composite flat panel for CCSS condition under bi-axial loading

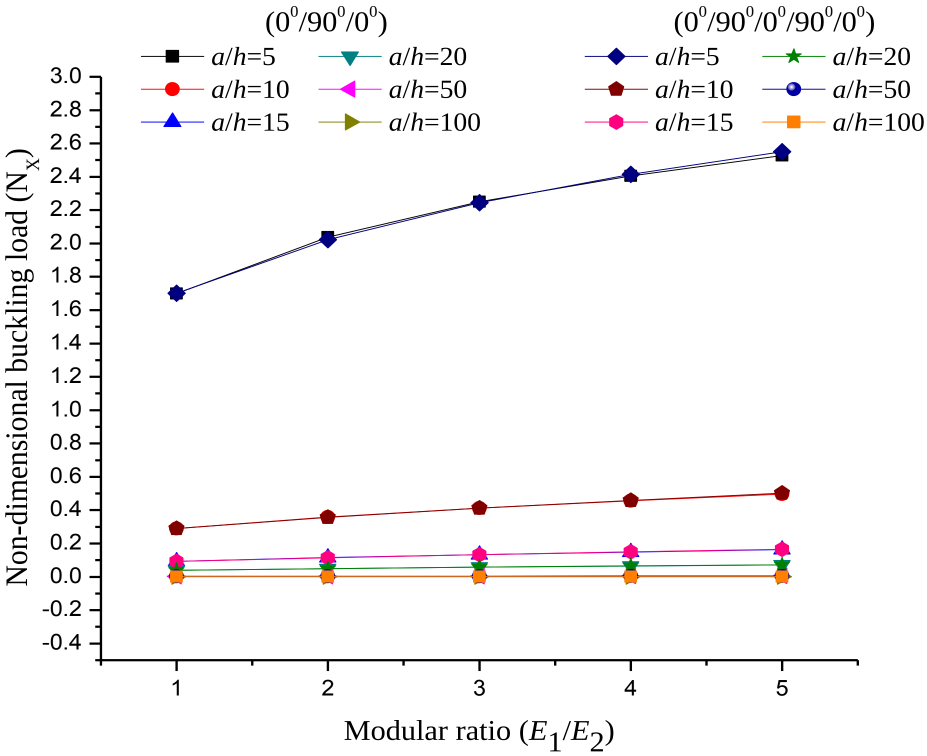


Figure 4.11 Non-dimensional buckling load of laminated composite flat panel for CCSS condition under uniaxial loading

Now, the analysis is extended for square laminated cross-ply $(0^0/90^0)_2$ and angle-ply $(\pm45^0)_2$ cylindrical panel under uniaxial loading. The non-dimensional mechanical buckling load parameters are obtained for four curvature ratios (R/a = 2, 5, 10 and 20) and five thickness ratios (a/h = 5, 10, 20 50 and 100) with two support condition (SSSS and CSCS) using material properties same as Nguyen-Van *et al*. [17]. The responses of cross-ply and angle-ply are presented in Figure 4.12 and Figure 4.13, respectively. It is noted that the buckling load parameter decreases with increase in thickness ratios and curvature ratios. It is well known that as the thickness ratio (a/h) increases the structure becomes thin and the panel becomes flat with increase in the curvature ratio (R/a) hence, the responses are in the expected line.

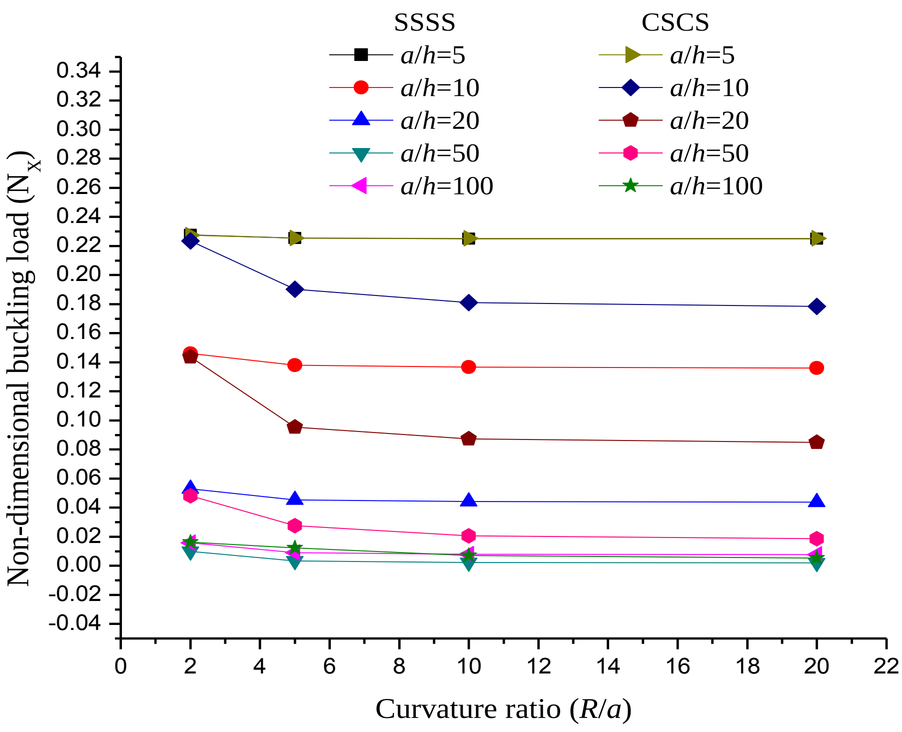


Figure 4.12 Non-dimensional buckling load of cross-ply $(0^0/90^0)_2$ laminated composite cylindrical panel for different support condition under uniaxial loading

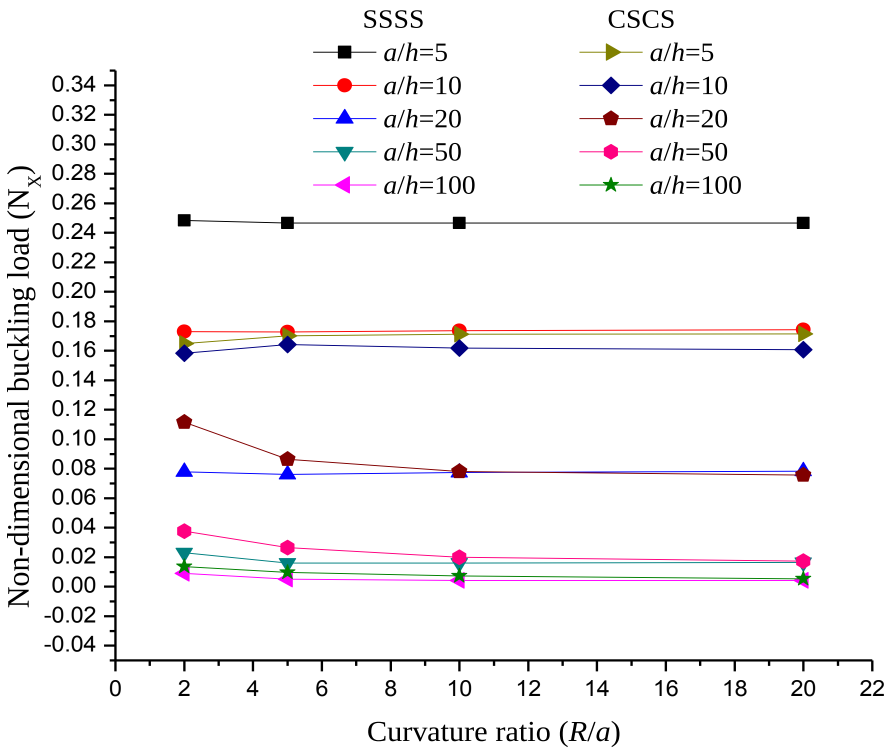


Figure 4.13 Non-dimensional buckling load of angle-ply $(\pm45^0)_2$ laminated composite cylindrical panel for different support condition under uniaxial loading

4.5.5 Thermo-Mechanical buckling analysis using HSDT model

Finally, thermo-mechanical buckling behaviour of laminated composite panels has been investigated using the present developed mathematical model. The non-dimensional critical buckling temperature parameters ($\lambda_{cr} = \alpha_0 T_{cr} 10^3$) have been obtained for square laminated cylindrical and spherical panels. For the computation purpose two clamped angle-ply lay-ups [$(\pm45^0)_S$ and $(\pm45^0)_2$] have been considered under the uniform temperature and the uniaxial mechanical loadings. The responses are plotted in the Figure 4.14 and Figure 4.15. It is observed that the critical buckling temperature parameter increases as the thickness ratio increases.

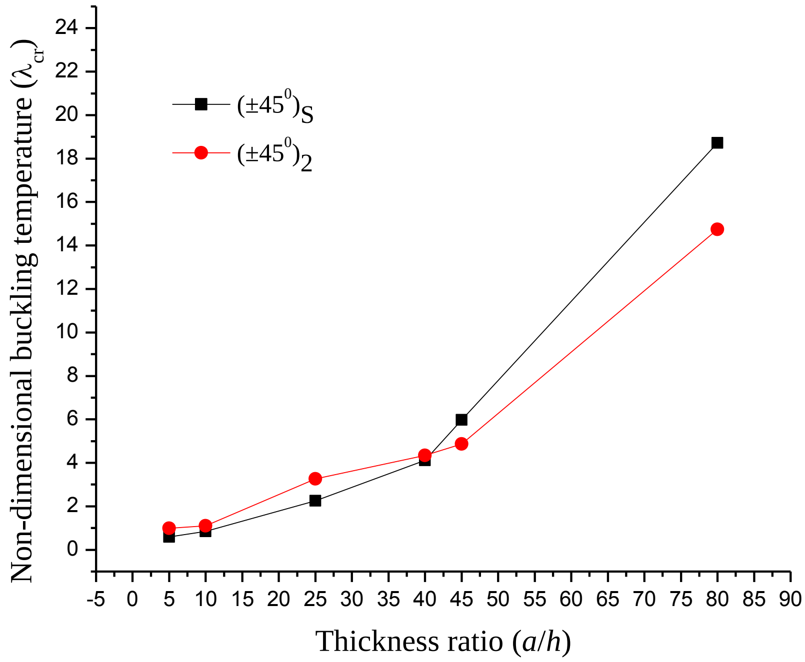


Figure 4.14 Non-dimensional critical buckling temperature of clamped square laminated composite cylindrical panel (*R*/*a*=10)

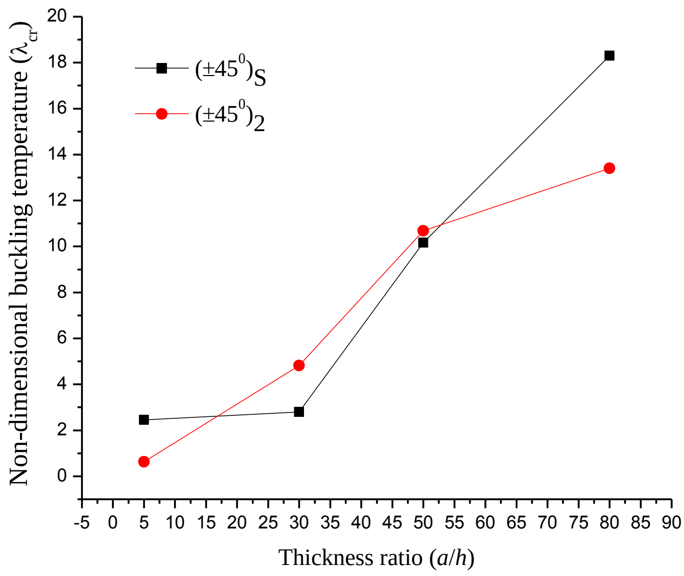


Figure 4.15 Non-dimensional critical buckling temperature of clamped square laminated composite spherical panel (R/a=10)

4.6 Conclusions

The thermal buckling strength of the laminated composite shallow shell panels subjected to uniform temperature and/or uniaxial or bi-axial mechanical loading is being examined by taking the nonlinearity in the geometry in Green-Lagrange sense. In addition to this, the excess thermal and/or mechanical deformation and the large rotation terms have been considered in the present formulation in the framework of the HSDT kinematics. A FEM model is developed and implemented for the discretisation of the mathematical model. Initially, the critical buckling temperature/load parameters are obtained by solving the linear eigenvalue problems using the steps in Chapter 2. Finally, the buckling temperature/load parameters for the different geometries of shell panels are computed for different combination of parameters such as the modular ratio, the lamination scheme, the thickness ratio and various support conditions. Based on the above numerical experimentation the following conclusions are drawn:

- The comparison study of the thermal/mechanical buckling load parameter indicates the necessity of the present Green-Lagrange type of nonlinearity for the evaluation of the geometry matrix.
- The parametric study indicates that the thermal/mechanical buckling load parameters greatly depended on the composite properties, support conditions, lamination schemes (cross-ply or angle-ply) and type of loading (bi-axial or uniaxial).

- The thermal/mechanical buckling load parameters increases with increase in the modular ratio with all types of support condition.
- The critical buckling temperature parameter is higher for angle-ply laminates in comparison to cross-ply laminates.
- Increase in the lay-up scheme has also significant effect on the critical buckling temperature parameters and it is noted that the temperature increases with increase in the number of layers.
- It is also interesting to note that in some of the cases of the thickness ratio the desired responses are reverted from their expected line. This is because of the fact that the thin structure may not follow a monotonous trend as expected for small strain and large (finite) deformation regime.
- From the study, it is understood that there is an importance of large rotation and deformation terms to consider in the mathematical model for highly flexible structures. In addition to that the geometric alternations especially laminated structures are need to be modelled using Green-Lagrange type of nonlinearity in the framework of the HSDT.

CHAPTER 5

CLOSURE

5.1 Concluding Remarks

Free vibration and buckling (thermal/mechanical/thermo-mechanical) behaviour of laminated composite single/doubly curved panels are computed using higher order mid-plane kinematics. The geometry matrix associated in buckling is evaluated using Green-Lagrange strain displacement relations to evaluate the excess thermal and/or mechanical deformation of laminated panel. A linear finite element is proposed and implemented to discretise the domain and solved numerically to obtain the desired responses by using a nine noded isoparametric Lagrangian element having ten degrees of freedom per node. The non-dimensional fundamental frequency and critical buckling load parameters are obtained solving the linear eigenvalue problem. The more specific conclusions as a result of the present investigation are stated below:

- In this present analysis Green-Lagrange type strain displacement relation is considered for the evaluation of geometric stiffness matrix for buckling analysis. The geometrical nonlinearity arising in the curved panel due to excess in-plane deformation have been considered to account the exact panel flexure. It is understood that the geometric nonlinearity modelled in von-Karman sense based on the FSDT and/or the HSDT is unrealistic in nature for the structures having severe geometric alteration.
- The system governing equations are derived by minimising the total potential energy and solved to obtain the desired responses. The model has been validated by comparing the responses with those published literature and comprehensiveness of the model is shown by solving different examples through the developed model. In addition to that the results are also computed in ANSYS using APDL code.
- The effect of various panel geometries (cylindrical, spherical, elliptical, hyperboloid and flat) and other parameters (thickness ratios, curvature ratios, aspect ratios, modular ratios, lay-up schemes and support conditions) on the free vibration behaviour are computed for without temperature effect. It is noted that the thickness ratio, the curvature ratio, the modular ratio, the aspect ratio and the edge support conditions are seen to affect the non-dimensional fundamental frequency parameter considerably.

- The non-dimensional fundamental frequency parameter increases with increase in the thickness ratio, the modular ratio and the number of layers, whereas it decreases with increase in the curvature ratio. The frequencies are not following same type of trend for lamination scheme and support conditions.
- The buckling (thermal/mechanical/thermo-mechanical) strength of laminated composite cylindrical, spherical, elliptical, hyperboloid and flat panels have been examined by taking the uniform temperature field throughout the thickness.
- Effect of the thickness ratios, the curvature ratios, the modular ratios, the lay-up schemes, the loading types (uniaxial/bi-axial) and the support conditions are studied in detail. It is observed that the non-dimensional critical buckling temperature parameter of angle-ply lamination are showing higher critical buckling temperature in comparison to the cross-ply laminations.
- The non-dimensional buckling load/parameter increases with increase in the modular ratio, the number of layers and the thickness ratio but it is important to mention that in some of the cases the responses are following a reverse trend for the small strain and large deformations. The responses are decreases with increase in the curvature ratio and non-monotonic behaviour is noted for the support conditions and the lay-up schemes.

5.2 Significant Contribution of the Thesis

The contributions of the present research work are as follows:

- A general mathematical model of curved panel has been developed in the framework of the HSDT mid-plane kinematics by taking all the higher order terms in the mathematical formulation for more accurate prediction of vibration and buckling (thermal/mechanical/thermo-mechanical) behaviour of laminated composite panels.
- In buckling case, Green-Lagrange type strain displacement relation is considered to take into account the geometrical nonlinearity arising in the curved panel due to excess thermal and/or mechanical deformation for evaluation of geometry stiffness matrix.
- Further, the panel model has been developed in the commercial FE software ANSYS using APDL code. Different numerical examples have been considered to show the efficacy of the developed mathematical and simulation model. The convergence and comparison study for free vibration and buckling behaviour of laminated panel is presented.
- A finite element method is employed for the discretisation purpose by using a nine noded isoparametric Lagrangian element with ten DOFs per node. In addition to this,

the panel geometry is developed and discretized in ANSYS using a Shell281 element (eight noded isoparametric serendipity shell element).

- The effects of various panel geometries (spherical, cylindrical, hyperboloid, elliptical and flat) and other geometrical parameters (thickness ratios, curvature ratios, aspect ratios, modular ratios, lamination schemes and support conditions) on free vibration responses are studied.
- Buckling (thermal/mechanical/thermo-mechanical) strength of the laminated composite spherical, cylindrical, hyperboloid, elliptical and flat panel is obtained by taking the uniform temperature throughout the thickness. The temperature independent composite material properties are taken for the present analysis. The critical buckling load parameters are obtained by solving the eigenvalue problem for different geometries of shell by taking the geometry nonlinearity in Green-Lagrange sense.

Finally, it is understood from the previous discussions that the developed general mathematical panel model in the framework of the HSDT would be useful for more accurate analysis of laminated composite structures exposed to vibration and/or excess thermal/mechanical/thermo-mechanical deformations. It is important to mention that the geometry matrix associated in the buckling has been taken in Green-Lagrange sense for the analysis. On the other hand, it is observed that the present developed FE model in ANSYS environment is also capable to solve any free vibration and buckling problem easily and with less computational time. And hence, the present analysis would be useful for practical design of the structure.

5.3 Future Scope of the Research

- The present study has been done by using the linear mathematical model only which can be extended for nonlinear analysis of laminated composite and sandwich structures.

- The present study can be extended to investigate the nonlinear free/forced vibration and thermo-mechanical post-buckling behavior of laminated composite/sandwich structures by taking temperature dependent material properties based on nonlinear mathematical model.

- An experimental study on vibration and buckling of laminated composite panels will gives better understanding about the present developed numerical model.

- By extending the present model, a nonlinear mathematical model can be developed to study the behaviour of laminated composite and sandwich structures in thermal and/or hygro-thermal environment.
- The smart (piezo, shape memory alloys and magnetostrictive) materials can be incorporated in the nonlinear model to study the effects of material and geometrical parameters.
- It will be interesting to study the flutter characteristics considering the aerodynamic and acoustic loading that arises frequently in the practical cases.

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Appendix

Appendix A. Nonlinear Strain Terms $\{\bar{\varepsilon}_{nl}\}$ and Thickness Co-ordinate Matrix as Appeared in General Mathematical Formulation

Nonlinear strain terms as shown in the Eq. (2.4)

$$u_{,x}=\frac{\partial u_0}{\partial x}+\frac{w_0}{R_x},u_{,y}=\frac{\partial u_0}{\partial y},\ v_{,x}=\frac{\partial v_0}{\partial x},v_{,y}=\frac{\partial v_0}{\partial y}+\frac{w_0}{R_y},\ w_{,x}=\frac{\partial w_0}{\partial x}-\frac{u_0}{R_x},\ w_{,y}=\frac{\partial w_0}{\partial y}-\frac{v_0}{R_y},$$

$$\theta_{x,x}=\frac{\partial\theta_x}{\partial x},\ \theta_{x,y}=\frac{\partial\theta_x}{\partial y},\theta_{y,x}=\frac{\partial\theta_y}{\partial x},\ \theta_{y,y}=\frac{\partial\theta_y}{\partial y},\theta_{z,x}=\frac{\partial\theta_z}{\partial x},\theta_{z,y}=\frac{\partial\theta_z}{\partial y},$$

$$\phi_{x,x}=\frac{\partial\phi_x}{\partial x},\phi_{x,y}=\frac{\partial\phi_x}{\partial y},\phi_{y,x}=\frac{\partial\phi_y}{\partial x},\phi_{y,y}=\frac{\partial\phi_y}{\partial y},$$

$$\psi_{x,x}=\frac{\partial\psi_x}{\partial x},\psi_{x,y}=\frac{\partial\psi_x}{\partial y},\psi_{y,x}=\frac{\partial\psi_y}{\partial x},\psi_{y,y}=\frac{\partial\psi_y}{\partial y},$$

$$(\varepsilon_x^4)=[(u_{,x})^2+(v_{,x})^2+(w_{,x})^2],(\varepsilon_y^4)=[(u_{,y})^2+(v_{,y})^2+(w_{,y})^2],$$

$$(\varepsilon_{x,y}^4)=2[u_{,x}u_{,y}+v_{,x}v_{,y}+w_{,x}w_{,y}],$$

$$(k_x^5)=2\left[\theta_{x,x}u_{,x}+\theta_{y,x},v_{,x}+\theta_{z,x}w_{,x}-\frac{\theta_x}{R_x}w_{,x}+\frac{\theta_z}{R_x}u_{,x}\right],$$

$$(k_y^5)=2\left[\theta_{x,y}u_{,y}+\theta_{y,y},v_{,y}+\theta_{z,y}w_{,y}-\frac{\theta_y}{R_y}w_{,y}+\frac{\theta_z}{R_y}v_{,y}\right],$$

$$(k_{x,y}^5)=2\left[\theta_{x,y}u_{,x}+\theta_{x,x}u_{,y}+\theta_{y,x}v_{,y}+\theta_{y,y}v_{,x}-\frac{\theta_y}{R_y}w_{,x}-\frac{\theta_x}{R_x}w_{,y}\right],$$

$$(k_x^6)=\left[\theta_{x,x}^2+\theta_{y,x}^2+2\phi_{x,x}u_{,x}+2\phi_{y,x}v_{,x}+\frac{\theta_x^2}{R_x{}^2}-\frac{\phi_x}{R_x}w_{,x}+\phi_{z,x}^2-2\theta_{z,x}\frac{\theta_x}{R_x}+2\theta_{x,x}\frac{\theta_z}{R_x}+\frac{\theta_z^2}{R_x^2}\right],$$

$$(k_y^6)=\left[\theta_{x,y}^2+\theta_{y,y}^2+2\phi_{x,y}u_{,y}+2\phi_{y,y}v_{,y}+\frac{\theta_y^2}{R_y^2}-\frac{\phi_y}{R_y}w_{,y}+\frac{\theta_x}{R_x}\frac{\phi_x}{R_x}\right],$$

$$(k_{x,y}^{6})=2\begin{bmatrix}\theta_{x,x}\theta_{x,y}+\theta_{y,x}\theta_{y,y}+\phi_{x,x}u_{,y}+\phi_{x,y}u_{,x}+\phi_{y,x}v_{,y}+\phi_{y,y}v_{,x}-\dfrac{\phi_x}{R_x}w_{,y}-\dfrac{\phi_y}{R_y}w_{,x}+\\ \dfrac{\theta_x}{R_x}\dfrac{\theta_y}{R_y}+\theta_{x,y}\dfrac{\theta_z}{R_x}+\theta_{y,x}\dfrac{\theta_z}{R_y}+\theta_{z,x}\theta_{z,y}-\dfrac{\theta_y}{R_y}\theta_{z,x}-\dfrac{\theta_x}{R_x}\theta_{z,y}\end{bmatrix},$$

$$(k_{x}^{7})=2\left[u_{,x}\psi_{x,x}+v_{,x}\psi_{y,x}+\theta_{x,x}\phi_{x,x}+\theta_{y,x}\phi_{y,x}-\frac{\psi_x}{R_x}w_{,x}+2\phi_{x,x}\frac{\theta_z}{R_x}-2\theta_{z,x}\frac{\phi_x}{R_x}+\frac{\theta_x}{R_x}\frac{\phi_x}{R_x}\right],$$

$$(k_{y}^{7})=2\left[u_{,y}\psi_{x,y}+v_{,y}\psi_{y,y}+\theta_{x,y}\phi_{x,y}+\theta_{y,y}\phi_{y,y}-\frac{\psi_y}{R_y}w_{,y}+2\phi_{y,y}\frac{\theta_z}{R_y}-2\theta_{z,y}\frac{\phi_y}{R_y}+\frac{\theta_y}{R_y}\frac{\phi_y}{R_y}\right],$$

$$(k_{x,y}^{7})=2\begin{bmatrix}u_{,x}\psi_{x,y}+u_{,y}\psi_{x,x}+v_{,x}\psi_{y,y}+v_{,y}\psi_{y,x}+\theta_{x,x}\phi_{x,y}+\theta_{x,y}\phi_{x,x}+\theta_{y,x}\phi_{y,y}+\theta_{y,y}\phi_{y,x}-\\ \dfrac{\theta_x}{R_x}w_{,y}-\dfrac{\psi_y}{R_y}w_{,x}+\dfrac{\phi_x}{R_x}\dfrac{\theta_y}{R_y}+\dfrac{\phi_y}{R_y}\dfrac{\theta_x}{R_x}+\phi_{x,y}\dfrac{\theta_z}{R_x}+\phi_{y,x}\dfrac{\theta_z}{R_y}-\theta_{z,y}\dfrac{\phi_x}{R_x}-\theta_{z,x}\dfrac{\phi_y}{R_y}\end{bmatrix},$$

$$(k_{x}^{8})=\left[\phi_{x,x}^{2}+\phi_{y,x}^{2}+2\theta_{x,x}\psi_{x,x}+2\theta_{y,x}\psi_{y,x}+\frac{\phi_x^2}{R_x^2}+2\frac{\theta_x}{R_x}\frac{\psi_x}{R_x}+\psi_{x,x}\frac{\psi_z}{R_x}-2\theta_{z,x}\frac{\psi_x}{R_x}\right],$$

$$(k_{x}^{8})=\left[\phi_{x,x}^{2}+\phi_{y,x}^{2}+2\theta_{x,x}\psi_{x,x}+2\theta_{y,x}\psi_{y,x}+\frac{\phi_x^2}{R_x^2}+2\frac{\theta_x}{R_x}\frac{\psi_x}{R_x}+\psi_{x,x}\frac{\psi_z}{R_x}-2\theta_{z,x}\frac{\psi_x}{R_x}\right],$$

$$(k_{y}^{8})=\left[\phi_{x,y}^{2}+\phi_{y,y}^{2}+2\theta_{x,y}\psi_{x,y}+2\theta_{y,y}\psi_{y,y}+\frac{\phi_y^2}{R_y^2}+2\frac{\theta_y}{R_y}\frac{\psi_y}{R_y}+\psi_{y,y}\frac{\psi_z}{R_y}-2\theta_{z,y}\frac{\psi_y}{R_y}\right],$$

$$(k_{x,y}^{8})=\begin{bmatrix}\phi_{x,x}\phi_{x,y}+\phi_{y,x}\phi_{y,y}+2\psi_{x,x}\theta_{x,y}+2\psi_{y,x}\theta_{y,y}+2\psi_{x,y}\theta_{x,x}+2\psi_{y,y}\theta_{y,x}+\\ 2\dfrac{\phi_x}{R_x}\dfrac{\phi_y}{R_y}+2\dfrac{\theta_x}{R_x}\dfrac{\psi_y}{R_y}+2\dfrac{\theta_y}{R_y}\dfrac{\psi_x}{R_x}+\psi_{x,y}\dfrac{\theta_z}{R_x}+\psi_{y,x}\dfrac{\theta_z}{R_y}-\theta_{z,y}\dfrac{\psi_x}{R_x}-\theta_{z,x}\dfrac{\psi_y}{R_y}\end{bmatrix},$$

$$(k_{x}^{9})=2\left[\phi_{x,x}\psi_{x,x}+\phi_{y,x}\psi_{y,x}+\frac{\phi_x}{R_x}\frac{\psi_x}{R_x}\right],(k_{y}^{9})=2\left[\phi_{x,y}\psi_{x,y}+\phi_{y,y}\psi_{y,y}+\frac{\phi_y}{R_y}\frac{\psi_y}{R_y}\right],$$

$$(k_{x,y}^{9})=2\left[\phi_{x,x}\psi_{x,y}+\phi_{y,x}\psi_{y,x}+\phi_{x,y}\psi_{x,x}+\phi_{y,y}\psi_{y,x}+\frac{\phi_y}{R_y}\frac{\theta_x}{R_x}+\frac{\phi_x}{R_x}\frac{\psi_y}{R_y}\right],$$

$$(k_x^{10}) = \left[\psi_{x,x}^2 + \psi_{y,x}^2 + \frac{\psi_x^2}{R_x^2} \right], (k_y^{10}) = \left[\psi_{x,y}^2 + \psi_{y,y}^2 + \frac{\psi_y^2}{R_y^2} \right],$$

$$(k_{x,y}^{10}) = 2\left[\psi_{x,x}\psi_{x,y} + \psi_{y,x}\psi_{y,y} + \frac{\psi_x}{R_x}\frac{\psi_y}{R_y} \right] \tag{A.1}$$

Linear and Non-linear Thickness co-ordinate Matrices as Appeared in Eq. (2.4)

$$[T_l] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 \end{bmatrix}$$

$$[T_{nl}] = \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 & 0 & 0 & z^4 & 0 & 0 & z^5 & 0 & 0 & z^6 & 0 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 & 0 & 0 & z^4 & 0 & 0 & z^5 & 0 & 0 & z^6 & 0 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 & 0 & 0 & z^4 & 0 & 0 & z^5 & 0 & 0 & z^6 \end{bmatrix}$$

(A.2)

Appendix B. Geometric Strain Vector and the Material Property Matrix Derivation

The expressions of geometric strain vector $\left[\bar{\varepsilon}_{nl}\right]$ and material property matrix $\left[D_G\right]$ in the main text are derived as

$$\left[\bar{\varepsilon}_{nl}\right]=\frac{1}{2}\begin{Bmatrix}\left[(u_{,x})^2+(v_{,x})^2+(w_{,x})^2\right]\\ \left[(u_{,y})^2+(v_{,y})^2+(w_{,y})^2\right]\\ 2\left[(u_{,x})(u_{,y})+(v_{,x})(v_{,y})+(w_{,x})(w_{,y})\right]\end{Bmatrix}$$

$$\text{or}\left[\bar{\varepsilon}_{nl}\right]=[T_{nl}][\varepsilon_{nl}]=[T_{nl}][A][G] \tag{B.1}$$

The values of $\left[A\right]$ and $\left[G\right]$ are

$$\left[A\right]=\begin{Bmatrix}\left[(u_{,x})+(v_{,x})+(w_{,x})\right]\\ \left[(u_{,y})+(v_{,y})+(w_{,y})\right]\\ \left[(u_{,x})(u_{,y})+(v_{,x})(v_{,y})+(w_{,x})(w_{,y})\right]\end{Bmatrix}\text{ and }\left[G\right]=\begin{bmatrix}u_{0_{,x}}\\ u_{0_{,y}}\\ v_{0_{,x}}\\ v_{0_{,y}}\\ w_{0_{,x}}\\ w_{0_{,y}}\end{bmatrix}$$

The values of material property matrix are obtained by the following procedure:

$$\left[D_G\right]=\sum_{k=1}^{N}\int_{h_{k-1}}^{h_k}\left[T_{nl}\right]^T\left\{\bar{S}\right\}^k\left[T_{nl}\right] \tag{B.2}$$

Nodal shape function $\left[N_i\right]$ is

$$\left[N_i\right]=\begin{bmatrix}N_i&0&0&0&0&0&0&0&0&0\\ 0&N_i&0&0&0&0&0&0&0&0\\ 0&0&N_i&0&0&0&0&0&0&0\\ 0&0&0&N_i&0&0&0&0&0&0\\ 0&0&0&0&N_i&0&0&0&0&0\\ 0&0&0&0&0&N_i&0&0&0&0\\ 0&0&0&0&0&0&N_i&0&0&0\\ 0&0&0&0&0&0&0&N_i&0&0\\ 0&0&0&0&0&0&0&0&N_i&0\\ 0&0&0&0&0&0&0&0&0&N_i\end{bmatrix}_{i=1\,to\,9} \tag{B.3}$$

Appendix C. Individual terms of the matrix [G]

$$[G]_{1\_1}=\frac{\partial}{\partial x},[G]_{1\_3}=\frac{1}{R_x},[G]_{2\_1}=\frac{\partial}{\partial y},[G]_{3\_2}=\frac{\partial}{\partial x},[G]_{4\_2}=\frac{\partial}{\partial y},[G]_{4\_3}=\frac{1}{R_y},$$

$$[G]_{5\_1}=-\frac{1}{R_x},[G]_{5\_3}=\frac{\partial}{\partial x},[G]_{6\_2}=-\frac{1}{R_y},[G]_{6\_3}=\frac{\partial}{\partial y},[G]_{6\_3}=\frac{\partial}{\partial y},[G]_{7\_4}=\frac{\partial}{\partial x},$$

$$[G]_{8\_4}=\frac{\partial}{\partial y},[G]_{9\_5}=\frac{\partial}{\partial x},[G]_{10\_5}=\frac{\partial}{\partial y},[G]_{11\_6}=\frac{\partial}{\partial x},[G]_{12\_6}=\frac{\partial}{\partial y},[G]_{13\_7}=\frac{\partial}{\partial x},$$

$$[G]_{14\_7}=\frac{\partial}{\partial y},[G]_{15\_8}=\frac{\partial}{\partial x},[G]_{16\_8}=\frac{\partial}{\partial y},[G]_{17\_9}=\frac{\partial}{\partial x},[G]_{18\_9}=\frac{\partial}{\partial y},[G]_{19\_10}=\frac{\partial}{\partial x},$$

$$[G]_{20\_10}=\frac{\partial}{\partial y},[G]_{21\_4}=1,[G]_{22\_5}=1,[G]_{23\_6}=1,[G]_{24\_7}=1,[G]_{25\_8}=1,[G]_{26\_9}=1,$$

$$[G]_{27\_10}=1 \tag{C.1}$$

About the Author

Katariya Pankajkumar Vaikunthbhai, the author, was born on 24th May 1990 in Naliya, Kachchh district, Gujarat, India. He has passed the Higher Secondary (10+2) Examination under the GSHEB, Gujarat with distinction in 2007 and graduated in Aeronautical Engineering with a first class degree from Sardar Vallabhbhai Patel Institute of Technology, Vasad, Gujarat, India in the year 2011. On completion of his graduation, he planned to continue his career in the field of research and joined M.Tech (Research) Program in NIT, Rourkela, Odisha, India with specialization in Machine Design and Analysis in 2012. The author's current research interest lies in the field of composite structures and finite element analysis.

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